

# On the efficiency of labor markets with short-time work policies

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## Abstract

This paper evaluates the social optimality of labor markets with search frictions and a job retention policy, named short-time work (STW), which has been applied in developed economies during the Covid-19 crisis. In a general equilibrium model, costs related to the STW cannot be internalized in wages, creating a systematic inefficiency through which job creation is too low. Government transfers to redistribute output to correct the inefficiency are proposed in the model. Furthermore, a calibration exercise matching the German economy over the period 2000-2021 suggests that transfers required for the social optimality are 1.9% of output. In addition, the unemployment rate is 1.8 percentage points lower in the presence of optimal transfers.

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# 1 Introduction

Short-time work (STW) is a policy program by which employers can temporarily reduce the working time of full-time workers. During the Covid-19 recession 2020-2021, governments in developed economies supported an unprecedented participation in the STW schemes, in two dimensions. First, the number of workers in STW was larger than ever before. For instance in Germany, 19% of employed workers were in STW during Spring 2020, compared with 4% during the Great Recession (Scarpetta, Pearson, Hijzen, & Andrea, 2020) and an average of 0.78% over the past 20 years.<sup>1</sup> Second, the reduction of working hours also reached a record level. More specifically, the working time of German workers in STW was reduced on average by almost 50% during the Covid-19, while it was by less than 30% during the Great Recession (Herzog-Stein, Nüß, Peede, & Stein, 2021).

Governments are subsidizing the participation in STW by providing workers a compensation of hours not worked, which usually corresponds to unemployment benefits. In addition, firms receive subsidies, such as the reimbursements of social security contributions, in order to incentivize higher participation in STW programs.<sup>2</sup> The goal of these policies is to retain jobs, in order to preserve the value of the job relationships and to stabilize unemployment.

The economic literature has indeed found that STW participates in preventing job destructions and limiting unemployment increase during recessions (Boeri and Bruecker (2011), Aiyar and Dao (2021), Cahuc, Kramarz, and Nevoux (2021), Kopp and Siegenthaler (2021)), but so far relatively little is known about the optimal level of these policies. This paper contributes to this gap by asking i) what is the socially efficient level of STW, and ii) under which conditions a competitive economy can reach this optimum. In order to answer these questions, I build on the decentralized general equilibrium search and matching labor market model by Balleer, Gehrke, Lechthaler, and Merkl (2016) and compare it to a constraint-efficient social-planner solution. I also introduce two government

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<sup>1</sup>Source: Bundesbank (<https://www.bundesbank.de/en/statistics>) and author's calculations.

<sup>2</sup> See section 3 for more details of the institutional framework.

subsidies and evaluate their capacity to restore the decentralized efficiency.

The intuition is as follows. Jobs are subject to an idiosyncratic productivity shock. With low values of this shock, workers may generate profit losses. In this case, a firm has two alternatives. First, it can participate in STW and decrease the working hours of low productive workers. Second, it can endogenously separate from these workers. If the firm chooses separation, it loses the value of a filled job, which arises from both costly vacancy creation and a fixed separation cost. If the net value of a job occupied by worker in STW is positive, then the firm participates in STW and keeps the worker employed. In addition, the extent by which the working time is reduced in STW is also costly. Without this cost, the firm would always reduce the working hours to zero in order to avoid any loss without losing the value of a filled job. Consequently, separations would never occur, which is counterfactual. Finally, the costs related to STW and separation have an impact on job creation. When a firm considers opening a vacancy, it has an expectation about the idiosyncratic productivity and, as a result, the STW and separation costs. Therefore, the firm opens less vacancies in the presence of these costs.

This paper considers a collective wage bargaining, in which the outside option is a strike. During a strike, job relationships are preserved, but STW and separation costs still hold.<sup>3</sup> Hence, in general the firm cannot internalize these costs in the collective bargaining. Therefore they create a systematic inefficiency in a competitive economy. In turn, the inefficiency has two consequences. First, competitive firms are creating less than the socially optimal amount of new jobs. Second, a competitive level of STW is lower than the socially optimal level. Both of these are a result of wages which are systematically too high. If the STW and separation costs would be internalized, the workers would obtain lower wages.

The social optimality can be restored under three conditions. The first is the so-called Hosios rule (Hosios, 1990), which is related to vacancy creation. More specifically, each new vacancy has a negative externality on the other firms and a positive externality on the

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<sup>3</sup>In an individual bargaining, workers' reservation wage would include the costs, since the workers avoid them in their outside option of unemployment.

unemployed workers. The Hosios rule states that these search externalities are internalized in wages if and only if the elasticity of matching function equals the bargaining power of workers. In addition, I show that two other conditions define the size of government transfers to firms. These transfers correct the socially inefficient output sharing of the competitive wage bargaining outcome. In other words, the transfers cover a fraction of STW and separation costs. More precisely, a STW transfer must equal the bargaining power of workers times the cost of working hour reduction, and a separation transfer must equal the bargaining power times the separation cost.

The different effects of the two transfers are investigated in a numerical exercise. At the steady-state, the STW transfer increases the hour reduction in STW and job creation simultaneously. On the contrary, the separation transfer only increases job creation without a change in the working hours in STW. The reason is that, the firm chooses the working time based on the job-specific productivity and wage. Furthermore, the firm is aware of this optimal working time when it decides about endogenous separations. Consequently, the working time choice affects separations, but not vice versa, and hence the working hours in STW are independent of the separation transfer.

Finally, the model is calibrated to match the German economy over the period of 2000-2021. In this calibration exercise, the transfers required for social optimality are found to be equal to 1.9% of output, which is substantial, considering that these transfers are the only subsidies in the model. As a comparison, in 2019, before the Covid-19 period, the labor market policy expenditures in Germany were 1.3% of GDP.<sup>4</sup> Furthermore, with the optimal transfers, the average unemployment rate would have been 6.0% instead of 7.8% without the transfers, suggesting a substantial improvement. In addition, the increased employment suggests fiscal benefits to finance the transfers.

The rest of the paper is organized as follows. Section 2 discusses the related literature. Section 3 describes the institutional framework of STW policies in Europe. Section 4 presents the model. Section 5 has the theoretical analysis. Section 6 has the quantitative

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<sup>4</sup>Source: European Commission, Directorate-General for Employment Social Affairs and Inclusion report, named Labour Market Policy - Expenditure and Participants, Data 2019, ISSN: 2467-4443.

analysis. Section 7 concludes.

## 2 Related literature

This paper investigates the social efficiency of labor markets with search and matching frictions. A fundamental rule of constraint efficiency in this class of models is derived by Hosios (1990) and further discussed in labor market context in Pissarides (2000). Hosios (1990) and Pissarides (2000) argue that competitive wage bargaining may not internalize externalities that vacancy postings have on the other searching firms and the unemployed workers in the same labor markets, and show that these congestion externalities are internalized in wages if and only if the elasticity of matching function equals the bargaining power of workers. In turn, if wages are determined by a wage-posting process instead of bargaining, and workers can direct their search towards specific wages, Moen (1997) and Acemoglu and Shimer (1999) show that the decentralized economy is always socially efficient in these so-called competitive search models.

In this paper, the model encompasses a Nash bargaining of wages and a random search. Consequently, the Hosios rule is necessary but not sufficient condition for optimality, due to the STW and separation costs. In this regard, this paper belongs to a large group of literature in which the optimality under the Hosios condition breaks down when additional features are included in the model. Some of the recent papers in this category are Flórez (2019) in which the inefficiency arises from addition of an informal sector, Cai (2020) with externalities from the on-the-job search, Albrecht, Cai, Gautier, and Vroman (2020) where the inefficiency is a consequence of workers sending multiple applications, Willems (2021) with externalities from a directed search, Laureys (2021) in which the inefficiency results from human capital depreciation, and Griffy and Masters (2022) with the inefficiency from participation externality.

To the best of my knowledge, the social optimality of labor markets with STW has received very little attention in the literature so far. Two early contributions are FitzRoy

and Hart (1985) and Burdett and Wright (1989). FitzRoy and Hart (1985) show that the implementation of payroll taxes, that finance an unemployment insurance can explain the choice between layoffs and working-time reductions, and argue that even though STW has an effect, it is quantitatively insufficient to explain the difference between the US and Europe. Burdett and Wright (1989) build a labor contract model, where wages and unemployment benefits are taxed with separate rates, and compare the unemployment insurance with a policy where also hours not worked are compensated. They show that the contract choice of agents is dictated by the ratio of the two tax rates, and that the policy choice can distort the economy to lower than full employment, or lower than full working time.

This paper also contributes to the theoretical literature of STW, including Boeri and Bruecker (2011), Niedermayer and Tilly (2016), Balleer et al. (2016) and Osuna and Pérez (2021). Boeri and Bruecker (2011) discuss the theory of STW in a stylized model where employment and hour supply choices are substitutes and conclude that the STW design should encourage hour adjustments during recessions, but may suffer from moral hazard in expansions. The importance of incentives for STW participation is also present in my paper. Niedermayer and Tilly (2016) build a partial equilibrium life-cycle model, where STW is modeled with flexible working hours and temporary unemployment. In turn, the model here is in general equilibrium, and STW has specific policy and cost functions. Osuna and Pérez (2021) have a model of STW with the Spanish characteristics, such as temporary work contracts and temporary layoffs, both of which are not considered here.

The model in this paper is based on Balleer et al. (2016). They use a dynamic stochastic general equilibrium model, to show that STW mitigates unemployment increase as a response to negative productivity shocks. Furthermore, they demonstrate that a discretionary change in STW policy does not have impact on unemployment, since the firms cannot anticipate the change. My paper builds on the decentralized model of Balleer et al. (2016), by deriving a social planner solution, and by adding government transfers in order to restore the decentralized efficiency.

So far, the research of STW has been mainly empirical. Abraham and Houseman (1994) use a linear model to estimate the employment and hours adjustment speed in the German, Belgian and French economies to a change in output, and compare it with the US. The Great Recession has inspired numerous articles. Hijzen and Venn (2011), Boeri and Bruecker (2011) do a cross-country analysis of OECD and developed economies. Balleer et al. (2016), Niedermayer and Tilly (2016) and Cooper, Meyer, and Schott (2017) use data from Germany, Cahuc et al. (2021) from France and Kopp and Siegenthaler (2021) from Switzerland. Later, Covid-19 recession caused an unprecedented take-up of STW policies and inspired a large quantity of research. These include Osuna and Pérez (2021) who analyze Spain, Aiyar and Dao (2021), Dengler and Gehrke (2021), Herzog-Stein et al. (2021) and Teichgräber, Žužek, and Hensel (2022) who research Germany, Benghalem, Cahuc, and Villedieu (2021) who investigate France and Hijzen and Salvatori (2022) who study Switzerland.

The empirical literature supports the contribution of STW in preventing job losses and stabilizing unemployment over business cycles, but also points out externalities including working hour distortions (Cooper et al., 2017), windfall effects, i.e. the heterogeneous effects of STW in different types of firms (Cahuc et al., 2021), deadweight costs, i.e. the excess cost of STW when the number of saved jobs is smaller than the number of workers participating in the program, (Boeri & Bruecker, 2011), and displacement effects, i.e. that STW maintains jobs which are not viable at all without subsidies (Osuna & Pérez, 2021). Many of these inefficiencies are related to the optimal level of STW, which this paper analyzes from a theoretical perspective.

### **3 Institutional framework**

This section discusses the STW and job retention programs in Europe, in order to present the important institutional features that the model in this paper is build to capture. STW policies have two important aspects. First, STW includes subsidies for firms and workers,

in order to incentivize the participation in the program. Consequently, the level of STW is affected by the generosity of the subsidies. Second, governments are setting criteria for the participation. Furthermore, the control of these criteria requires administrative effort from firms. In conclusion, the level of STW is a result of, on the one hand, the gains from participation in the program, and on the other hand, the costs and the limiting criteria of the participation.

The number of countries implementing STW jumped during the Covid-19 pandemic, but the policy itself has a long history, for instance in Germany and Switzerland where it has existed since the 1920s (Müller & Schulten, 2020). However, the coverage, eligibility and generosity of established STW programs have been adjusted during the large economic crises such as the Great Recession or the Covid-19 (Mosley, 2020b).

The main part of STW policies is a compensation paid to workers for hours not worked, since the workers are also earning only a fraction of their normal full wage. In general, this compensation is equivalent with an unemployment insurance, but can be more or less generous, having varied from a 100% replacement rate in Denmark and Netherlands to a 50% in Poland during the Covid-19 Müller and Schulten (2020). Most commonly, the government covers all the STW allowances from the unemployment insurance systems, social security funds, or from the government budget (Mosley, 2020b). However, some countries, e.g. Sweden and Italy, also require some fraction of the allowance from the employer. In addition, the payment method varies between countries. In most countries, STW compensation is paid as a wage allowance to the employer, who then pays it to the worker along with the part of the wage. However, for instance in Finland and Spain, the STW allowance is paid directly to the employee Müller and Schulten (2020).

The second type of STW-related transfers are subsidies to firms. The two main subsidies are a compensation of social security payments and a support for training for the workers in STW. These two can also be combined. For instance, in Spain the government subsidizes 50% of social security contributions of a worker in STW, which is increased



to 80% if the worker participates in training (Osuna & Pérez, 2021).<sup>5</sup> These subsidies decrease the cost of STW to firms, while the training maintains human capital during the hours not worked.

The criteria for STW vary considerably between different countries. Generally, there are criteria for major events that are covered by the STW program, most notably, a temporary economic downturn, a bad weather in construction industry, restructuring or force majeure. In addition, there are restrictions for both firms and workers. For the firms, these relate most commonly to the reduction in working time and the number of employees, but also to the firm size or business sectors. For the workers, for instance fixed-term contracts or recently hired workers may be excluded. (Mosley, 2020b)

The details of the administrative processes in different countries vary even more than the STW criteria. However, the main part is an application usually to some dedicated government office. The application process may require negotiations between the firm and the national or local authorities, trade-unions or employee representatives (Mosley, 2020b). In some cases, such as Germany, the workers' approval is required before their wages can be reduced in STW (Balleer et al., 2016).

The next section of this paper describes the model. For the modeling perspective, the institutional setting is summarized in two key points. First, the subsidies to firms encourage them to increase the level of STW above what they would choose when only considering the labor cost reductions. Second, the administrative process requires effort, making STW costly for the firms in a non-pecuniary sense. These costs along the criteria by the government limits the level of STW. Both of the two elements will be part of the model. On the one hand, the costs of STW, in both dimension, the number of workers and the number of working hours, are modeled as cost functions. On the other hand, there are transfers reducing these costs, which are also analyzed in order to correct the social inefficiencies in the model.

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<sup>5</sup>Workers can also receive higher STW allowances if they participate in training, e.g. in France 100% of hourly wage instead of 70% (Mosley, 2020b).

## 4 Model

The model is based on Balleer et al. (2016). The economy features labor markets with search and matching frictions and endogenous job destruction and creation processes à la Mortensen and Pissarides (1994). The labor market model is supplemented with a short-time work policy (STW). The government collects a lump-sum tax from households to finance transfers to firms, which are studied to correct the social inefficiencies from STW.

The economy encompasses households, firms and the government. Households provide labor, consume and save to government risk free bonds. Household members are either employed or unemployed. Firms post vacancies to hire unemployed workers to produce a homogeneous consumption good using labor as the productive input. Job productivity is subject to an idiosyncratic shock. For the low values of this shock firms can decide to destroy these jobs as in Mortensen and Pissarides (1994), or reduce the working hours by participating in STW. Full details of the model are in the appendix A.

### 4.1 The firm

Each job within a firm has an idiosyncratic productivity, which is drawn each period from a time-invariant distribution. In addition, all the jobs share a common productivity component. The total productivity of a job is a combination of the two, as (Details in Appendix A.1)

$$y_{i,t} = A_t - \varepsilon_{i,t}, \tag{1}$$

where  $y_{i,t}$  is output of job  $i$ ,  $A_t$  is total productivity, and  $\varepsilon_{i,t}$ , an idiosyncratic productivity. Based on the total productivity of the job, i.e.  $A_t - \varepsilon_{i,t}$  the workers are divided into three groups. The first group with the lowest productive workers are endogenously separated and become unemployed. The second group are the workers that are sent to STW, and the third group are the workers who work full-time.

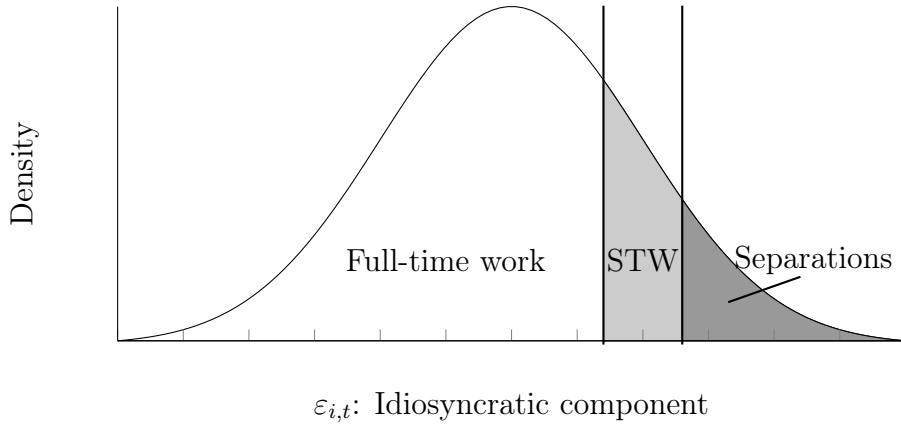


Figure 1: The job productivity is combined by a common and idiosyncratic components as  $A_t - \varepsilon_{i,t}$ . The latter is drawn from a time-invariant distribution. This draw divides workers in three groups: separations, STW, and full time work.

Figure 1 illustrates the idiosyncratic productivity draw and the division of workers into three groups. The endogenously separated workers become unemployed. Hence, the total employment stock, is a sum of employees in full-time work and in STW, given by

$$n_t = n_t^{FT} + n_t^{STW}, \quad (2)$$

where  $n_t$  is the total employment,  $n_t^{FT}$  the stock of workers working full-time and  $n_t^{STW}$  the stock of workers in STW. As a difference to the standard search and matching model, the employment evolution is presented separately for the two worker groups. The stock of full-time workers evolves as

$$n_t^{FT} = (1 - \rho^x)(1 - \rho_t^e)(1 - \chi_t) [n_{t-1}^{FT} + n_{t-1}^{STW} + q(\theta_{t-1})v_{t-1}], \quad (3)$$

where  $\rho^x$  is a rate of exogenous separations,  $\rho_t^e$  a rate of endogenous separations and  $\chi_t$  a rate of workers in STW,  $q(\theta_t)$  a vacancy filling rate and  $v_t$  the number of vacancies. As in the canonical employment evolution equations, the current period employment consists of

the previous period employed workers, here  $n_{t-1}^{FT} + n_{t-1}^{STW}$ , and of new matches,  $q(\theta_{t-1})v_{t-1}$ . The former means that the full-time workers can remain in the full-time stock, but also the previous period workers in STW can become full-time due to a new productivity draw.

Symmetrically, the stock of workers in STW, evolves as

$$n_t^{STW} = (1 - \rho^x)(1 - \rho_t^e)\chi_t [n_{t-1}^{FT} + n_{t-1}^{STW} + q(\theta_{t-1})v_{t-1}]. \quad (4)$$

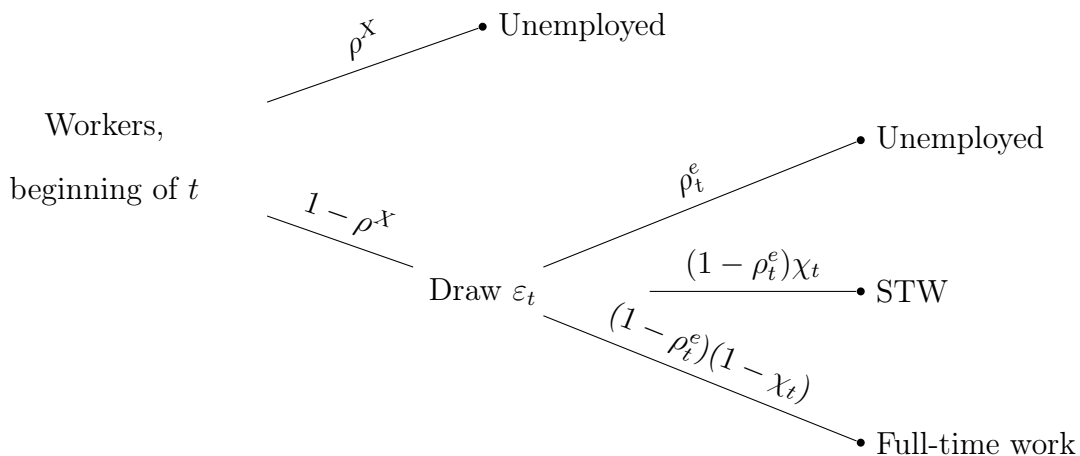


Figure 2: The sequence of separations and STW, where  $\rho^x$  and  $\rho_t^e$  are exogenous and endogenous separation rates and  $\chi_t$  STW rate. Exogenous separations occur first, followed by the idiosyncratic draw, which results in the firm's decision about endogenous separations and STW.

Again, the period  $t - 1$  full-time worker can be sent to STW in period  $t$ . The new matches from period  $t - 1$  are subject to the same transition probabilities as the existing matches and these workers can also be sent to STW or separate endogenously immediately in their first employment period.

Figure 2 depicts the probability tree of separation, STW and full-time work rates. The exogenous separations can be considered to occur first, after which the idiosyncratic productivity is drawn. The firm chooses endogenous separations and STW. The workers who are not endogenously separated nor sent to STW, work full time. Note that, this

definition gives a total separation rate as in Den Haan, Ramey, and Watson (2000), i.e.

$$\rho_t = \rho^X + (1 - \rho^X)\rho_t^e. \quad (5)$$

The firm's choice of sending workers to STW is considered in two dimensions, each one with a cost. The first is the number of workers sent to STW, captured by  $\chi_t$  above. A cost related to  $\chi_t$  is given by

$$X(\chi_t) = \frac{c_\chi}{2}\chi_t^2, \quad (6)$$

where  $c_\chi$  is a scale parameter. A quadratic cost function is chosen to allow an interior solution, as compared to a linear cost which would result in the firm sending either all or none of the workers to STW. The cost function  $X(\chi_t)$  can be interpreted as a policy criterion which the government imposes on the STW. For instance, a lower scale parameter  $c_\chi$  would imply more lax policy and increase the number of workers in STW.

The second dimension is the number of working hours reduced from the workers in STW. A cost in the number of hours is given by

$$C(K_t^*) = \frac{c_k - \tau^K}{2}(K_t^*)^2, \quad (7)$$

where  $K_t^*$  is an optimal hour reduction,  $c_k$  is a fixed cost and  $\tau^K$  is a STW transfer from the government. A full working time is normalized to one, and the hour reduction  $K_t^*$  is a fraction from the full time continuous between 0 and 1. Likewise with the cost function (6), the quadratic form here allows the optimal choices other than only 0 or 1.

The firm produces homogeneous consumption good using labor as the input of production. The profits of the firm are given by

$$\begin{aligned} \Pi_t = & \underbrace{n_t^{FT} A_t^{FT} + n_t^{STW} A_t^{STW} (1 - K_t^*)}_{\text{Total output}} - \underbrace{w_t^{FT} n_t^{FT} - w_t^{STW} n_t^{STW} (1 - K_t^*)}_{\text{Total wages}} \\ & - \underbrace{C(K_t^*) n_t^{STW} - X(\chi_t) - \frac{\rho_t^e}{1 - \rho_t^e} n_t (f - \tau^f) - \kappa v_t}_{\text{Costs and transfers}}, \end{aligned} \quad (8)$$

where  $A_t^{FT}$  and  $A_t^{STW}$  are aggregate outputs and  $w_t^{FT}$  and  $w_t^{STW}$  are aggregate wages of full-time and short-time workers respectively,  $f$  is a separation cost and  $\kappa$  a vacancy cost. The separation cost is paid for endogenously separated workers. It is important to notice that the existence of STW does not require separation cost  $f$ . This cost is added to allow calibrations to match relevant levels of STW. Since vacancy posting is costly, all preserving matches have a positive value and consequently some workers generating deficits would be sent to STW albeit the separation cost.

In addition, I consider an additional policy tool, namely a separation transfer  $\tau^f$ . This transfer, alike the STW transfer in the cost function (7), are considered here in order to compensate social inefficiencies due to these costs. Wages are bargained collectively, and they contain the productivity dependent component.

The firm chooses 6 control variables, namely the number of vacancies  $v_t$ , the number of workers in full-time work  $n_t^{FT}$ , the number of workers in STW  $n_t^{STW}$ , the hour reduction in STW  $K_t$ , the share of workers in STW  $\chi_t$  and the rate of endogenous separations  $\rho_t^e$  so as to maximize the expected discounted real profits as

$$\max_{v_t, n_t^{FT}, n_t^{STW}, K_t, \chi_t, \rho_t^e} E_t \sum_{t=0}^{\infty} \beta^t \left( \Pi_t \right), \quad (9)$$

subject to the employment evolution equations (2), (3), (4) and the STW cost functions (6) and (7).

The first-order conditions for vacancies,  $v_t$ , full-time workers,  $n_t^{FT}$ , and STW workers,  $n_t^{STW}$ , are combined to form a job-creation condition (JCC) as

$$\begin{aligned} \frac{\kappa}{q(\theta_t)} = & \beta E_t \left\{ (1 - \rho^x) \left[ \underbrace{(1 - \rho_{t+1}^e)(1 - \chi_{t+1})(A_{t+1}^{FT} - w_{t+1}^{FT})}_{\text{Profits of full-time work}} \right. \right. \\ & \left. \left. + \underbrace{(1 - \rho_{t+1}^e)\chi_{t+1}[(1 - K_{t+1}^*)(A_{t+1}^{STW} - w_{t+1}^{STW}) - C(K_{t+1}^*)]}_{\text{Profits of STW}} \right] \right. \\ & \left. - \underbrace{(1 - \rho^x)\rho_{t+1}^e(f - \tau^f)}_{\text{Separation costs}} + \underbrace{(1 - \rho_{t+1})\frac{\kappa}{q(\theta_{t+1})}}_{\text{Continuation value}} \right\} \end{aligned} \quad (10)$$

JCC equates the cost of hiring a new worker with the expected gains from a new job. The gains consist of expected full-time work and STW profits and continuation value, net of costs. The costs are related to hour reductions in STW, separations and vacancy postings. Note that, if there is no STW, i.e.  $\chi_{t+1} = 0$ , the model collapses to a canonical search and matching model. This can be seen from the job creation condition (10) where the second line disappears.

The first-order condition for hour cut  $K_t$  results in an optimal hour reduction condition given by

$$K_t^* = -\frac{A_t^{STW} - w_t^{STW}}{c_k - \tau^K}. \quad (11)$$

The worker who generates contemporaneous deficits, i.e.  $A_t^{STW} < w_t^{STW}$ , can be sent to STW. The optimal choice of  $K_t^*$  is negatively related to productivity and positively to wage. For instance, more productive workers work more hours in STW and higher wage leads to higher hour reductions. The cost  $c_k$  has negative impact on  $K_t^*$ , i.e. the higher the cost the smaller the working time cut. Transfer  $\tau^K$  is studied later as part of the optimal policy.

The first-order condition for the share of workers in STW,  $\chi_t$  is formulated as the following condition

$$\chi_t = (\lambda_t^{STW} - \lambda_t^{FT}) \frac{n_t}{c_\chi} \quad (12)$$

where  $\lambda_t^{FT}$  and  $\lambda_t^{STW}$  are Lagrange multipliers of constraints related to the full-time and the STW workers' evolution equations (3) and (4). It presents the marginal gain for the firm from having a worker in STW instead of full-time work. If sending a marginal worker to STW increases profits of the firm more than having her working full time, i.e.  $\lambda_t^{STW} > \lambda_t^{FT}$ , the share  $\chi_t$  increases and vice versa. The cost  $c_\chi$  for choosing the STW share is firm not worker specific and hence the total impact of the marginal gain is scaled by the size of employment  $n_t$ . One way to consider this is that the cost is an effort lost when firms negotiate about the  $\chi_t$  with the government. The negotiation process is the same for different sized firms, hence larger firms gain more from STW than smaller firms

and choose larger share  $\chi_t$ .

A first order condition for the share of endogenous separations is formulated as the following condition

$$\rho_t^e = \frac{f - \tau^f}{\lambda_t^{FT}(1 - \chi_t) + \lambda_t^{STW}\chi_t} + 1. \quad (13)$$

The firm takes into account the possibility to send low productive workers to STW, and reduce the deficits which they generate. Hence, the cost of separations, i.e.  $f - \tau^f$ , is relative to a change in the marginal values of full-time and STW workers. This equation is not interpreted or developed further here, but compared with the social planner condition in the analysis.

Wages are bargained collectively, which corresponds to many European countries for instance Germany (Balleer et al., 2016). A labor union bargains wages for all workers. An outside option is a strike. During the strike, workers remain matched with the firm, but no production occurs. The workers do not earn wages but the home production  $b$ . The firm also holds the match value during the strike. The union is assumed to bargain a merit based wage such that the more productive workers earn higher wages. The Nash bargaining problem is (Details in the appendix A.4)

$$\arg \max_{w_t} (W_t - \tilde{W}_t)^\gamma (F_t - \tilde{F}_t)^{1-\gamma}, \quad (14)$$

where  $\gamma$  is the bargaining power of workers,  $W_t$  the worker's value when working,  $\tilde{W}_t$  the workers value on strike,  $F_t$  the firms value of production and  $\tilde{F}_t$  the firms value of strike.

The solution results in a wage rule that is dependent on the idiosyncratic productivity. The job specific wage is given by

$$w_t = \gamma(A_t - \varepsilon_t) + (1 - \gamma)b. \quad (15)$$

The wage rule is aggregate for the firm's problem over the idiosyncratic component, to result the wages for the full-time and STW workers.



The above definition of wages fully internalizes the idiosyncratic productivity of jobs. This definition of wages is used to simplify the comparison of decentralized and social planner solutions. I acknowledge that this wage may in some cases result in an incentive incompatibility for workers in STW, even though the worker's quitting decision is not modeled in detail. However, this wage definition allows to pin down the social inefficiency from STW, even when the wages are adjusting to low productivities. As a comparison, Appendix A.5 considers the additional inefficiencies that arise from the idiosyncratic productivity when the same wage is paid to all workers.

## 4.2 The social planner

The socially optimal solution in this section follows a standard approach to constraint efficiency in markets with search and matching frictions, e.g. Pissarides (2000). The social welfare consists of household's consumption. An aggregate consumption is a sum of output and home production net of real costs. The social planner is subject to the same labor market frictions as the competitive firm, including the costs related to STW. The social planner optimizes welfare as

$$\begin{aligned}
 U_t^{SP} = \max_{v_t, n_t^{FT}, n_t^{STW}, K_t, \chi_t, \rho_t^e} E_t \sum_{t=0}^{\infty} \beta^t & \left[ \underbrace{n_t^{FT} A_t^{FT} + n_t^{STW} A_t^{STW} (1 - K_t)}_{\text{Output}} \right. \\
 & + \underbrace{b(1 - n_t + K_t n_t^{STW})}_{\text{Home production}} \\
 & \left. - \underbrace{C(K_t) n_t^{STW} - X(\chi_t) - \frac{\rho_t^e}{1 - \rho_t^e} n_t f - \kappa v_t}_{\text{Costs}} \right], \tag{16}
 \end{aligned}$$

subject to the same conditions as the firm (2), (3), (4), (6) and (7). The social planner problem does not include wages. Wages are only redistributing output between agents, and by assumption, the distributional considerations are excluded from the social welfare function. For the same reason, home production is considered as an outside option for workers instead of unemployment benefits. The unemployment benefits would also only

redistribute output without an effect on welfare.

The first-order conditions for vacancies,  $v_t$ , full-time workers,  $n_t^{FT}$ , and STW workers,  $n_t^{STW}$ , are combined to form a job-creation condition (JCC) as

$$\begin{aligned} \frac{\kappa}{q(\theta_t)(1-\xi_t)} = \beta E_t \left\{ (1-\rho^x) \left[ (1-\rho_{t+1}^e)(1-\chi_{t+1})(A_{t+1}^{FT} - b) \right. \right. \\ \left. \left. + (1-\rho_{t+1}^e)\chi_{t+1}[(1-K_{t+1}^*)(A_{t+1}^{STW} - b) - C(K_{t+1}^*)] \right] \right. \\ \left. - (1-\rho^x)\rho_{t+1}^e f + (1-\rho_{t+1}) \frac{\kappa}{q(\theta_{t+1})(1-\xi_{t+1})} \right\}, \end{aligned} \quad (17)$$

where  $\xi_t$  is defined as  $-\xi_t \equiv \frac{\theta_t q'(\theta_t)}{q(\theta_t)}$  and  $q'(\theta_t)$  is the derivative of  $q(\theta_t)$  w.r.t.  $\theta_t$ . More intuitively put,  $\xi_t$  is the elasticity of unemployment in the matching function. The social JCC is compared with the competitive JCC in the optimality analysis later.

The first-order condition for hour reduction  $K_t$  results in an optimal condition given by

$$K_t^* = -\frac{A_t^{STW} - b}{c_k}. \quad (18)$$

The home production  $b$  is directly present in the social hour reduction condition (18), as compared with the competitive condition (11). It has a positive impact on  $K_t^*$ . The reason is that the social optimality is a balance between the output and the home production. By the definition of social welfare function (16), the share of hours  $K_t^*$  that an STW worker is not working, she uses for home production. If, ceteris paribus,  $b$  increases, the household's utility from one hour of home production exceeds the same hour in output production. As a result, the socially optimal hour reduction increases.

The first-order condition for the share of workers in STW,  $\chi_t$  is formulated as

$$\chi_t = (\phi_t^{STW} - \phi_t^{FT}) \frac{n_t}{c_\chi}, \quad (19)$$

where  $\phi_t^{FT}$  and  $\phi_t^{STW}$  are the Lagrange multipliers of the constraints of employment evolutions (3) and (4). Likewise the JCC, this condition (19) is compared with the competitive solution in the optimality analysis. Same applies to an endogenous separation condition

which is formulated as

$$\rho_t^e = \frac{f}{\phi_t^{FT}(1 - \chi_t) + \phi_t^{STW} \chi_t} + 1. \quad (20)$$

## 5 Analysis

This section compares the competitive and social planner solutions analytically. The first part of the section shows that there are inefficiencies, and detects their size and sign. The second part proposes sufficient conditions to correct the inefficiencies in a decentralized economy. The optimality conditions include the transfers presented in the model, and this section also investigates the size of these transfers.

### 5.1 Size and sign of inefficiencies

In order to compare job creation conditions, the wage equations for full-time and STW workers are inserted into the decentralized JCC equation (10). Then the decentralized JCC (10) is subtracted from the social planner's condition (17) yielding (details in appendix A.3)

$$\begin{aligned} E_t \beta \frac{(1 - \rho_{t+1}) \kappa}{q(\theta_{t+1})} & \underbrace{\left( \frac{1}{1 - \xi_{t+1}} - \frac{1}{1 - \gamma} \right) - \left( \frac{1}{1 - \xi_t} - \frac{1}{1 - \gamma} \right)}_{\text{The congestion externality, (+/-)}} \frac{\kappa}{q(\theta_t)} \\ & - \underbrace{\left( f - \frac{f - \tau^f}{1 - \gamma} \right) \beta E_t [(1 - \rho^x) \rho_{t+1}^e]}_{\text{Inefficiency from separation cost } f, (-)} \\ & - \underbrace{\left( c_k - \frac{c_k - \tau^K}{1 - \gamma} \right) \frac{1}{2} E_t \beta (K_{t+1}^*)^2}_{\text{Inefficiency from STW cost } c_k, (-)} = 0. \end{aligned} \quad (21)$$

In this comparison of dynamic equations, the time-varying variables, such as the labor-market tightness  $\theta$  are assumed to be identical between the two solutions. Equation

(21) shows the three sources of inefficiency in the decentralized economy: the congestion externality, separation costs and STW costs. In general, the inefficiencies are negative and result in too little job creation, except if the congestion externality has considerably high positive effect.

The congestion externality is a familiar result from models with search and matching frictions. When a firm posts a vacancy, the labor-market tightness increases. Other firms which are searching become worse off, since vacancy filling becomes less likely. On the other hand, the unemployed workers who are searching for a job become better off, since they are more likely to match with an open vacancy. The wage bargaining between firms and workers may not internalize the impact that the vacancy posting has on labor markets, wages can be either too high or too low compared with the socially optimal outcome. As a result competitive firms end up creating too many or too few new jobs. Further on, unemployment is not on the socially optimal level.

The other inefficiencies from the separation cost and the STW cost have unarguably negative effect. When a firm is hiring a new worker, the outcome of the match is unknown, i.e. the idiosyncratic productivity has not realized yet. There is a chance that the idiosyncratic productivity is low, and the firm has to either send the worker to STW or, in the worst case, separate with the worker. Both of these actions are costly to the firm. Since, the competitive firm has an expectation about the idiosyncratic draw, it will create less jobs the higher these costs.

The STW and separation costs are not internalized in the wage bargaining. These costs materialize only if the firm chooses to send workers to STW or separate. Hence, the wage bargaining could only include the expectation of the costs. In the collective bargaining process, the outside option is a strike. When a strike occurs, the expected STW and separation costs remain, since they depend on the idiosyncratic draw which is independent from striking. As a result, the firm cannot bargain lower wages with these costs, because in the strike the costs would persist but no production occurs.<sup>6</sup> In the

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<sup>6</sup>More formally, the costs appear in both of the firm's value functions  $F_t$  and  $\tilde{F}_t$  and cancel each other when the two are deducted,  $F_t - \tilde{F}_t$ , in the Nash problem.

individual competitive bargaining the firm could deduct the costs from a wage, but even in this case, the worker would lose the cost when employed but avoid it when unemployed. Hence, the cost would increase the reservation wage as part of the worker's outside option in the bargaining process.

The size of inefficiencies depend on the bargaining power of workers and is expressed by the multipliers inside of the brackets in each line of equation (21). The size of congestion externality is dictated by the difference of elasticity term  $\xi_t$  and  $\xi_{t+1}$  and the bargaining power of workers  $\gamma$ . In search and matching models the matching function is often defined to have a constant elasticity for instance a Cobb-Douglas form, which makes comparison with constant bargaining power straightforward.

The separation cost and STW cost create a negative externality with an equal relative size. While the social planner only considers costs  $f$  and  $c_k$ , the competitive firm multiplies these costs with an inverse of the firm's bargaining power, i.e.  $1/1-\gamma$ , which by definition of the bargaining power is greater than 1. These costs cannot be internalized in wages and the firm bears all of them. When the workers have positive bargaining power, the firm only obtains fraction of the production surplus and has less resources to allocate to job creation. Higher the bargaining power, the more the expected STW and separation costs decrease the number of vacancies. Another difference between the socially optimal and competitive solutions are the transfers  $\tau^f$  and  $\tau^K$ , which are considered in the next section to correct the inefficiencies.

## 5.2 Optimal policy

The previous section detected social inefficiencies by comparing the social planner solution with the decentralized solution. As a result, the sources and sizes of externalities are determined in the model. Consequently, it is possible to solve conditions under which the decentralized economy can restore the social optimality. These conditions for social optimality are formulated in the following proposition.

**Proposition 1** *The decentralized level of job creation is socially optimal if the following three sufficient conditions hold simultaneously:  $\xi_t = \gamma, \forall t$ , i.e. the elasticity of matching function with respect to unemployment must equal the bargaining power of workers (Hosios condition),  $\tau^f = \gamma f$ , i.e. the separation transfer must equal worker's bargaining power times the separation cost and  $\tau^K = \gamma c_k$ , i.e. the STW transfer must equal worker's bargaining power times the unit cost of hour reduction.*

PROOF, appendix A.3

Proposition 1 contains three sufficient conditions for the optimality. The first condition for the optimality is the famous Hosios rule (see Hosios (1990) and Pissarides (2000)). It states that if and only if the bargaining power of workers equals the elasticity of matching function with respect to unemployment, the bargaining results in wages that are optimal by internalizing the impact of vacancy postings on the searching firms and workers on the same labor market.

The second condition is related to the separation costs  $f$ . The condition dictates that the optimal separation transfer is the bargaining power times the separation cost, i.e.  $\tau^f = \gamma f$ . Because the firm needs to bear all this cost, it will create less jobs when the cost exists. The government can increase job creations by subsidizing firms. The optimal transfer cancels out the bargaining outcome by redistributing to the firm a share of output relative to the bargaining power of workers. The optimal transfer enables the firm to create equal amount of new jobs than it would create when obtaining all the production surplus. This corresponds to the choice of the social planner, since the planner is optimizing output without redistribution considerations.

The third condition is related to the cost of STW. Analogically to the separation cost and transfer, the STW transfer must equal the bargaining power of workers times the unit cost of hour reduction, i.e.  $\tau^K = \gamma c_k$ . The competitive firm compares the cost of reducing working hours with the reduction of deficits, which the low productive worker generates, and decides the working time accordingly. Higher bargaining power means that the firm can reduce more wage costs when cutting the working time in STW, but it also

means that the profitable full-time workers obtain a larger share of output, leaving the firm with less recourse to use for fixed STW costs. The social planner only compares the utility gained from output with the utility from home production without distortions from surplus sharing. As a result, the social planner would allocate more to job creation and higher employment by reducing more working hours in STW, than the decentralized firms which consider the surplus sharing.

The above presented three conditions, the Hosios rule, the separation cost rule and the STW cost rule are sufficient but not necessary conditions for the constraint efficiency. Other combination of conditions may also lead to social optimality. From the sufficient conditions, it is rather straightforward to see that if no workers are in STW, this implies that the Hosios rule and the optimal separation transfer are sufficient to ensure the social optimality. As mentioned in the previous section, if there is no STW, the model collapses to a canonical search and matching model with the separation cost. If the separation cost does not exist, the Hosios condition alone is sufficient for the optimality.

Moreover, at least in a theoretical sense, the third option to reach the social optimality is based on the different signs of the externalities. The violation of Hosios condition can result in a positive or a negative externality, meaning too many or too few new vacancies are opened. On the contrary, the separation and STW costs are creating solely negative externalities. In theory, it is hence possible that a positive externality from the violation of Hosios condition is outweighed by the negative impact of separation and STW costs. This case is rather theoretical, and acknowledged here, but a quantification of the magnitudes of the different conditions are not studied further.

In addition, it is worth to emphasize that under the sufficient conditions, there are no other inefficiencies from STW and separation choices by the firm. More specifically, the choices which the firm makes, i.e. the vacancy posting, the share of workers in STW and the rate of separations, are conducted by comparing marginal costs of these choices. The marginal values consist of the profit functions for the competitive firm, which are equal for all the labor market choices, hence, also the inefficiencies in all choices are

equivalent. As a result, the sufficient conditions ensure the efficiency of vacancy creation, STW participation and separation decisions of competitive firms.

## 6 Quantitative analysis

This section is analyzing the empirical relevance of the model, with a calibration exercise matching quarterly values of German economy between 2000-2021. The German economy is chosen for three main reasons. First, labor market data including STW statistics are readily available. Second, the German STW program is well established, most researched and has inspired other European policymakers when they have implemented their own job retention policies. Third, the model in this paper builds on Balleer et al. (2016) which is primarily constructed to model the German labor markets allowing a cross-checking of the calibration.

### 6.1 Calibration exercise

The essential factor of social optimality are the costs of STW and separations. As these costs cannot be directly observed from the data, the first part of the calibration exercise is to detect the values of cost parameters in the model. In order to accomplish this task the steady-state of the model is calibrated to the quarterly values of German economy between 2000-2021.

The strategy is to choose the steady-state targets and parameter values, which have clear counterparts in the data, or established values in the literature. Based on these calibrated values, the model equations are used to detect the cost parameters of the model. Finally these costs then dictate the optimal transfers.

Table 1 summarizes the steady-state targets and calibrated parameters. Unemployment rate  $u$  is set to 7.8% and STW rate  $\chi$  0.78%, which are the averages from the German data between 2000 and 2021. The STW rate is slightly higher than in Balleer et al. (2016), who target a value 0.69%, due to the high rates during Covid-19. The



aggregate reduction in working hours  $K^*$  is one third from full working time, which is a long term average in Germany according to Balleer et al. (2016).

Parameter	Description	Value	Source
$\beta$	Discount factor	0.99	Annual risk-free rate 4%
$u$	Unemployment rate	0.078	Average 2000-2021
$m$	Matching efficiency	0.5	To match $q = 0.70$ and $v = 0.03$
$\mu$	Matching elasticity	0.5	$q = 0.70$ , $v = 0.03$ and the Hosios condition
$\chi$	Rate of STW	0.0078	Average 2000-2021
$K^*$	Working time reduction	0.33	Balleer et al. (2016)
$\gamma$	Workers' bargaining power	0.5	The Hosios condition
$\rho^X$	Exogenous separations	0.02	Balleer et al. (2016) and $v = 0.03$
$\rho^e$	Endogenous separations	0.011	Balleer et al. (2016)
$A$	TFP	1	Standard in the literature
$b$	Home production	0.75	Losses in STW profits in FT
$\sigma$	STD of idiosyncratic distribution	1.65	Losses in STW profits in FT

Table 1: Calibration

The matching function parameters, the elasticity  $\mu = 0.5$  and the efficiency  $m = 0.5$  are calibrated to target a vacancy filling rate close to 0.7 and a vacancy rate to 0.03, which both follow Balleer et al. (2016). In addition, the bargaining power of workers is set equal to the matching elasticity to make the Hosios condition bind. The bargaining power value of 0.5 is symmetric between agents, and hence uninformative, which also makes it a common value in the literature.

The total separation rate is approximated to 3% in Balleer et al. (2016), with a deviation of 1/3 in endogenous and 2/3 in exogenous rates. I choose a slightly higher endogenous rate of 0.011 in order to increase the vacancy filling rate closer to the target. The total

productivity  $A = 1$  is standard in the literature. The home production is set to  $b = 0.75$ . Finally, the idiosyncratic productivity is chosen as normally distributed with mean 0 and standard deviation of  $\sigma = 1.65$ . Both home production and idiosyncratic productivity are calibrated to result in the STW workers to generate losses while the full-time workers generate profits simultaneously. In addition, home production is a flow value of unemployment to workers capturing the utility from all outside of work activities. That is why the value of  $b$  is considered higher than unemployment insurance alone would imply.

Parameter	Description	Value	Source
$n$	Employment	0.922	$1 - u$ , labor force normalized to 1
$\rho$	Total separation rate	0.0308	$\rho = \rho^X + (1 - \rho^X)\rho^e$
$v$	Vacancies	0.044	Steady-state matches and matching function
$\theta$	Tightness	0.56	$v/u$
$q(\theta)$	Vacancy filling rate	0.67	Matching function calibration
$c_k$	Working-time reduction cost	10.55	The cost function $C(K)$
$c_\chi$	STW-share scalar parameter	-367.32	The cost function $X(\chi)$
$f$	Separation cost	0.84	Job destruction condition
$\kappa$	Vacancy cost	1.45	Matching function

Table 2: Calibration implied values

In addition to these calibrated parameters, I implement the wage rule by Balleer et al. (2016) to the calibrated model (see details in Appendix A.5). This wage is chosen for two reasons. First, it is better in line with the German economy. Second, it makes workers in STW more expensive for firms incentivizing more endogenous separations. The latter effect is necessary later when the optimal transfers are imposed in the model. Otherwise, the flexible wages for STW workers together with the optimal transfers would imply zero, or even negative endogenous separation rates, with plausible calibration values, making a

comparison of different cases impossible.<sup>7</sup>

Table 2 presents the values which are implied by the calibration in Table 1 and the model equations. The vacancy rate  $v$  is higher than the target, but on the contrary and the vacancy filling rate  $q$  is slightly lower. These are still in acceptable level, given that the rest of the calibration is feasible with these values. Essential parameters for the analysis are the costs  $c_k$ ,  $c_\chi$  and  $f$ . These values are arbitrary, and their sizes cannot be interpreted as such, but they are used to indicate the optimal transfers. However, the scalar  $c_\chi$  of the cost function of the STW rate may seem unusual as it is negative. The intuition is that it captures a government's STW policy criterion, i.e. the magnitude of losses which the worker needs to generate before participation in STW is accepted.

The results from the calibration exercise show that the transfers required for the socially optimality are 1.9% of output. As a comparison, all labor market expenditures in Germany during 2019 were 1.3% of GDP.<sup>8</sup> Since there are no other labor market subsidies in the model, the relative size of STW and separation transfers can be considered realistic. Even though, in reality, the share of STW related subsidies from the total labor market expenditures is relatively small, for instance an average was 2.94% between 2007-2017 in Germany (Mosley, 2020a). In addition, the model fails to capture this feature as the relative size of STW transfer is approximately 3 times larger than the separation transfer.

## 6.2 Calibration of the optimal policy

The next part of the calibration exercise uses the costs which were solved above. These costs are now taken as given, and related optimal transfers are applied in the model. The new steady states are solved given the transfers, both jointly and separately. Table 3 summarizes the steady-state results.

In the presence of optimal transfers, the steady-state unemployment rate decreases

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<sup>7</sup>The other option could be to model the endogenous quitting decision of workers. In which case, low wages would increase total separations through increase in quits. I leave this extension for future research.

<sup>8</sup>Source: European Commission, Directorate-General for Employment Social Affairs and Inclusion report, named Labour Market Policy - Expenditure and Participants, Data 2019, ISSN: 2467-4443.

significantly from 7.8% to 6%. At the same time, the level of STW is higher as the steady-state hour reduction increases from one third to approximately three fourth of the full working time. As a consequence of increase in STW, the separation rate drops from 3.1% to 2.3%. More surprisingly, the share of workers in STW, i.e.  $\chi$  is lower with the optimal policy. The reason is that both, the higher hour cut, and the transfers subsidizing costs, result in higher profitability of the firm. Consequently, the firm is able to have more STW workers back to full-time work. Simultaneously, more low productive workers can be employed in STW in stead of separation. However, the former group of workers is larger than the latter one, leading to a lower STW rate.

Variable	Long name	No policy	Optimal policy	Only $\tau^K$	Only $\tau^f$
$u$	Unemployment	0.078	0.060	0.050	0.100
$v$	Vacancies	0.044	0.033	0.026	0.062
$\theta$	Tightness	0.564	0.547	0.516	0.620
$q$	Job filling rate	0.666	0.676	0.716	0.635
$\rho$	Separations	0.031	0.023	0.020	0.042
$\chi$	STW rate	0.0078	0.0068	0.0080	0.0070
$K^*$	Hour cut	0.33	0.77	0.85	0.29
$Y$	Output	0.971	0.970	0.965	0.982
$C$	Consumption	0.966	0.968	0.967	0.966

Table 3: Comparison of steady-states with optimal transfers

Another, perhaps unexpected result is a decrease in the vacancy rate, which falls from 4.4% with the baseline calibration to 3.3% with the optimal transfers. However, the steady-state labor market tightness and consequently the job filling rate remain relatively close to their original values. The former decreases from 0.56 to 0.55 and latter increases from 0.67 to 0.68. These two variables capture the general state of labor markets, which remains close between the two steady states. As a consequence, the considerably lower unemployment rate is accompanied by a lower vacancy rate, to yield relatively corresponding

labor market tightness in the presence of the optimal policy.

Finally, the results about output and consumption confirm that the optimal policy indeed is welfare enhancing. Even though, output slightly decreases when the optimal transfers are applied, consumption is still higher than without the policy, due to home production. Thus, in general, the optimal policy divides labor input more efficiently between output and home production, than in the decentralized equilibrium.

Table 3 also has the steady states when the two transfers are applied separately. The previous analysis of different channels how the transfers affect the economy is confirmed, as the two steady states differ significantly. Unsurprisingly, the STW transfer  $\tau^K$ , increases the hour reduction, since it decreases the related cost. Consequently, the lower productive workers can be kept employed in STW, and the separation rate decreases. More interestingly, the unemployment rate falls even more than with the optimal policy, as the firm is also able to keep more workers employed in full-time work, due to a higher profitability.

The steady state with only separation transfer  $\tau^f$  has also many expected results. Because the separation costs are lower with the transfer, the separation rate increases. Even though, the higher separation rate is accompanied by considerably higher vacancy rate, the latter is not sufficient to prevent unemployment from becoming higher than in the baseline calibration, due to a higher labor market tightness and a lower vacancy filling rate. As expected, the separation transfer alone does not cause much changes in the level of STW. Especially the hour reduction decreases only by 4 percentage points.

As a conclusion, the two transfers give incentives for firms to optimize their production in a different way. The STW transfer subsidizes labor hoarding, i.e. the firms optimize output by keeping a large share of workers employed, by also cutting a large fraction of working hours from those participating in STW. In turn, the separation transfer makes the firms cut a large share of low productive workers in order to create fewer new high productive jobs instead. Both of the transfers have their pros and cons. The STW transfer results in low unemployment but low productivity as well. On the contrary, the separation

transfer induces high unemployment accompanied with high productivity. In general, the calibration exercise suggests that labor market policies should be designed as a whole, since individual subsidies may have desired effects on some dimensions, but unexpected tradeoffs in the others.

## 7 Conclusion

This paper studies in a theoretical context, the social optimality of labor markets, with search frictions and short-time work policy. Workers with low productivity may generate contemporaneous profit losses, in which case firms can participate to STW and reduce working hours. However, participation in STW is costly to the firms because the administrative process requires effort and governments are regulating the STW programs. Due to these costs, the firms in the model of this paper are creating too few new jobs, and the level of STW is too low from a social point of view. This paper proposes transfers in the model to redistribute output to firms in order to correct the inefficiencies.

European policies, which for instance subsidize social security contributions and offer training for workers, are aiming to increase the level of STW especially during economic downturns. The transfers in this paper can be considered to capture a part of these existing policy measures in many European countries. The transfers are able to restore the social efficiency of decentralized economy, which suggests that the European governments are aware of the potential shortfalls in the level of STW without additional incentives.

The STW programs aim to prevent excess job losses and unemployment fluctuations in particular during recessions, and are shown to contribute in this task. This paper considers one potential trade-off of the policy, lower job creations. Indeed, when the firms are optimizing their output production keeping current employees in STW, the investments in new jobs can be neglected. Another potential trade-off is information asymmetry and moral hazard. The asymmetric information is a consequence of firms' ability to observe the profitability of their employees, which the government is unable

to detect. Consequently, setting the efficient STW-policy criterion and subsidies is a complicated task, and may end up in an overly generous policy. Hence, the firm may be tempted to send profitable workers to STW in order to collect excess profits on the expense of the government. The model of this paper contains elements such as job specific productivities, government transfers and STW criteria, that allow the model with feasible extensions to address also these drawbacks of STW. I leave this analysis for future research.

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# A Appendix, Model derivation

This section contains the details of the derivation of the model, the decentralized and competitive solutions and their comparisons. In addition, an alternative wage rule and its efficiency is discussed in subsection A.5.

## A.1 Aggregation and notation

The productivity of each job consists of a common component which is the same for all jobs, and an idiosyncratic component which is job specific. The output of a job  $i$  with idiosyncratic productivity  $\varepsilon_{i,t}$  is given by

$$y_{i,t} = A_t - \varepsilon_{i,t}, \quad (\text{A.1})$$

where  $A_t$  is common productivity component, and  $\varepsilon_{i,t}$  is drawn from a time-invariant distribution with PDF  $g(\varepsilon)$ .

The detailed conditions for endogenous separations and STW are derived later, both of which induces a threshold value of idiosyncratic productivity. These thresholds are namely STW threshold  $v_t^k$  and separation threshold  $v_t^f$ . Workers with idiosyncratic productivity  $\varepsilon_t < v_t^k$  are normal full-time workers. For the clarity of expression, an aggregate output of full-time workers is named  $A_t^{FT}$ . It is a conditional mean of the idiosyncratic component, conditioned on the STW threshold, given by

$$A_t^{FT} = \int_{-\infty}^{v_t^k} A_t - \varepsilon_t g(\varepsilon) d\varepsilon. \quad (\text{A.2})$$

Respectively, the aggregate output of STW workers is named  $A_t^{STW}$ , and given by

$$A_t^{STW} = \int_{v_t^k}^{v_t^f} A_t - \varepsilon_t g(\varepsilon) d\varepsilon. \quad (\text{A.3})$$

In other words, workers with idiosyncratic productivity  $v_t^k < \varepsilon_t < v_t^f$  are in STW.

Using the thresholds above the share of endogenously separated workers becomes

$$\rho_t^e = \int_{v_t^f}^{\infty} g(\varepsilon) d\varepsilon, \quad (\text{A.4})$$

in which  $\rho_t^e$  denotes the endogenous separation rate. Respectively, the rate of workers in STW, named  $\chi_t$  is given by

$$\chi_t = \int_{v_t^k}^{v_t^f} g(\varepsilon) d\varepsilon. \quad (\text{A.5})$$

### A.1.1 Firm's problem

A firm maximizes intertemporal profits by

$$\begin{aligned} \max_{v_t, n_t^{FT}, n_t^{STW}, K_t, \chi_t, \rho_t^e} E_t \sum_{t=0}^{\infty} \beta^t & \left( n_t^{FT} A_t^{FT} + n_t^{STW} A_t^{STW} (1 - K_t^*) \right. \\ & - w_t n_t^{FT} - w_t n_t^{STW} (1 - K_t^*) \\ & \left. - C(K_t^*) n_t^{STW} - X(\chi_t) - \frac{\rho_t^e}{1 - \rho_t^e} n_t (f - \tau^n) - \kappa v_t \right), \end{aligned} \quad (\text{A.6})$$

such that

$$\begin{aligned} n_t^{FT} &= (1 - \rho^x) [(1 - \rho_t^e)(1 - \chi_t)(n_{t-1}^{FT} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{STW})], \\ n_t^{STW} &= (1 - \rho^x) [(1 - \rho_t^e)\chi_t(n_{t-1}^{STW} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{FT})], \\ n_t &= n_t^{FT} + n_t^{STW} \\ X(\chi_t) &= \frac{c_\chi}{2} \chi_t^2 \\ C(K_t^*) &= \frac{c_k - \tau^K}{2} (K_t^*)^2, \end{aligned} \quad (\text{A.7})$$

FOCs w.r.t  $v_t$ ,  $n_t^{FT}$ ,  $n_t^{STW}$ ,  $\chi_t$  and  $\rho_t^e$  with Lagrange multipliers  $\lambda^{FT}$  and  $\lambda^{STW}$  for employment evolution constraints:

$$\begin{aligned} -\kappa + \beta E_t \lambda_{t+1}^{FT} (1 - \rho^x) (1 - \rho_{t+1}^e) (1 - \chi_{t+1}) q(\theta_t) \\ + \beta E_t \lambda_{t+1}^{STW} (1 - \rho^x) (1 - \rho_{t+1}^e) \chi_{t+1} q(\theta_t) &= 0 \end{aligned} \quad (\text{A.8})$$

$$A_t^{FT} - w_t - \frac{\rho_t^e}{1 - \rho_t^e}(f - \tau^n) - \lambda_t^{FT} \quad (\text{A.9})$$

$$+ \beta E_t \lambda_{t+1}^{FT} (1 - \rho^x)(1 - \rho_{t+1}^e)(1 - \chi_{t+1}) + \beta E_t \lambda_{t+1}^{STW} (1 - \rho^x)(1 - \rho_{t+1}^e) \chi_{t+1} = 0$$

$$A_t^{STW} (1 - K_t^*) - w_t (1 - K_t^*) - C(K_t^*) - \frac{\rho_t^e}{1 - \rho_t^e}(f - \tau^n) - \lambda_t^2 \quad (\text{A.10})$$

$$+ \beta E_t \lambda_{t+1}^{FT} (1 - \rho^x)(1 - \rho_{t+1}^e)(1 - \chi_{t+1}) + \beta E_t \lambda_{t+1}^{STW} (1 - \rho^x)(1 - \rho_{t+1}^e) \chi_{t+1} = 0$$

$$- \lambda_t^{FT} (1 - \rho^x)(1 - \rho_t^e)(n_{t-1}^{FT} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{STW}) + \quad (\text{A.11})$$

$$\lambda_t^{STW} (1 - \rho^x)(1 - \rho_t^e)(n_{t-1}^{STW} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{FT}) - c_\chi \chi_t = 0$$

$$- \frac{(f - \tau^f)n_t}{(1 - \rho_t^e)^2} - \lambda_t^{FT} (1 - \rho^x)(1 - \chi_t)(n_{t-1}^{FT} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{STW}) \quad (\text{A.12})$$

$$- \lambda_t^{STW} (1 - \rho^x) \chi_t (n_{t-1}^{STW} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{FT}) = 0$$

Intermediate results:

$$\frac{\kappa}{q(\theta_t)} = \beta E_t \lambda_{t+1}^{FT} (1 - \rho^x)(1 - \rho_{t+1}^e)(1 - \chi_{t+1}) + \beta \lambda_{t+1}^{STW} (1 - \rho^x)(1 - \rho_{t+1}^e) \chi_{t+1},$$

$\Rightarrow$

$$\lambda_t^{FT} = A_t^{FT} - w_t - \frac{\rho_t^e}{1 - \rho_t^e}(f - \tau^f) + \frac{\kappa}{q(\theta_t)} \quad (\text{A.13})$$

and

$$\lambda_t^{STW} = A_t^{STW} (1 - K_t^*) - w_t (1 - K_t^*) - C(K_t^*) - \frac{\rho_t^e}{1 - \rho_t^e}(f - \tau^f) + \frac{\kappa}{q(\theta_t)}$$

Resulting in job creation condition as

$$\begin{aligned}
\frac{\kappa}{q(\theta_t)} &= \beta E_t \left( A_{t+1}^{FT} - w_{t+1} - \frac{\rho_{t+1}^e}{1 - \rho_{t+1}^e} (f - \tau^f) + \frac{\kappa}{q(\theta_{t+1})} \right) \\
&\quad (1 - \rho^x)(1 - \rho_{t+1}^e)(1 - \chi_{t+1}) \\
&+ \beta E_t \left( A_{t+1}^{STW} (1 - K_{t+1}^*) - w_{t+1}(1 - K_{t+1}^*) - C(K_{t+1}^*) - \right. \\
&\quad \left. \frac{\rho_{t+1}^e}{1 - \rho_{t+1}^e} (f - \tau^f) + \frac{\kappa}{q(\theta_{t+1})} \right) \\
&\quad (1 - \rho^x)(1 - \rho_{t+1}^e)\chi_{t+1}
\end{aligned} \tag{A.14}$$

$\Leftrightarrow$

$$\begin{aligned}
\frac{\kappa}{q(\theta_t)} &= \beta E_t \left[ (1 - \rho_{t+1}) \left\{ (1 - \chi_{t+1})(A_{t+1}^{FT} - w_{t+1}) \right. \right. \\
&\quad \left. \left. + \chi_{t+1}[(1 - K_{t+1}^*)(A_{t+1}^{STW} - w_{t+1}) - C(K_{t+1}^*)] \right\} \right. \\
&\quad \left. - (1 - \rho^x)\rho_{t+1}^e(f - \tau^n) + (1 - \rho_{t+1})\frac{\kappa}{q(\theta_{t+1})} \right]
\end{aligned}$$

Deriving  $\chi_t$

$$\begin{aligned}
& -\lambda_t^{FT}(1 - \rho^x)(1 - \rho_t^e)(n_{t-1}^{FT} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{STW}) + \\
& \lambda_t^{STW}(1 - \rho^x)(1 - \rho_t^e)(n_{t-1}^{STW} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{FT}) - c_\chi \chi_t = 0 \\
& \Leftrightarrow \\
c_\chi \chi_t &= (\lambda_t^{STW} - \lambda_t^{FT})(1 - \rho^x)(1 - \rho_t^e)(n_{t-1}^{FT} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{STW}) \\
& \Leftrightarrow \\
c_\chi \chi_t &= (\lambda_t^{STW} - \lambda_t^{FT})(1 - \rho_t)(n_{t-1}^{FT} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{STW}) \\
& \Leftrightarrow \\
\chi_t &= (\lambda_t^{STW} - \lambda_t^{FT}) \frac{n_t}{c_\chi}
\end{aligned} \tag{A.15}$$

Deriving  $\rho_t^e$

$$\begin{aligned}
& -\frac{(f - \tau^f)n_t}{(1 - \rho_t^e)^2} - \lambda_t^{FT}(1 - \rho^x)(1 - \chi_t)(n_{t-1}^{FT} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{STW}) \\
& \quad - \lambda_t^{STW}(1 - \rho^x)\chi_t(n_{t-1}^{STW} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{FT}) = 0 \\
& \Leftrightarrow \\
& \frac{(f - \tau^f)n_t}{(1 - \rho_t^e)^2} = -\lambda_t^{FT}(1 - \rho^x)(1 - \chi_t)(n_{t-1}^{FT} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{STW}) \\
& \quad - \lambda_t^{STW}(1 - \rho^x)\chi_t(n_{t-1}^{STW} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{FT}) \\
& \Leftrightarrow \\
& \frac{(f - \tau^f)n_t}{1 - \rho_t^e} = -\lambda_t^{FT}(1 - \rho^x)(1 - \rho_t^e)(1 - \chi_t)(n_{t-1}^{FT} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{STW}) \\
& \quad - \lambda_t^{STW}(1 - \rho^x)(1 - \rho_t^e)\chi_t(n_{t-1}^{STW} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{FT}) \\
& \Leftrightarrow \\
& \frac{(f - \tau^f)n_t}{1 - \rho_t^e} = -[\lambda_t^{FT}(1 - \chi_t) + \lambda_t^{STW}\chi_t](1 - \rho^x)(1 - \rho_t^e)(n_{t-1}^{STW} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{FT}) \\
& \Leftrightarrow \\
& \frac{(f - \tau^f)n_t}{1 - \rho_t^e} = -[\lambda_t^{FT}(1 - \chi_t) + \lambda_t^{STW}\chi_t]n_t \\
& \Leftrightarrow \\
& f - \tau^f = -[\lambda_t^{FT}(1 - \chi_t) + \lambda_t^{STW}\chi_t](1 - \rho_t^e) \\
& \Leftrightarrow \\
& \frac{f - \tau^f}{\lambda_t^{FT}(1 - \chi_t) + \lambda_t^{STW}\chi_t} + 1 = \rho_t^e
\end{aligned} \tag{A.16}$$

Applying the definitions of  $\lambda_t^{FT}$  and  $\lambda_t^{STW}$ , the condition can be developed further as

$$\begin{aligned}
\frac{(f - \tau^f)}{1 - \rho_t^e} &= -[\lambda_t^{FT}(1 - \chi_t) + \lambda_t^{STW} \chi_t] \\
&\Leftrightarrow \\
\frac{(f - \tau^f)}{1 - \rho_t^e} &= -\left\{ \left[ A_t^{FT} - w_t - \frac{\rho_t^e}{1 - \rho_t^e} (f - \tau^f) + \frac{\kappa}{q(\theta_t)} \right] (1 - \chi_t) + \right. \\
&\left. \left[ A_t^{STW} (1 - K_t^*) - w_t (1 - K_t^*) - C(K_t^*) - \frac{\rho_t^e}{1 - \rho_t^e} (f - \tau^f) + \frac{\kappa}{q(\theta_t)} \right] \chi_t \right\} \\
&\Leftrightarrow \\
\frac{(f - \tau^f)}{1 - \rho_t^e} &= -\left\{ [A_t^{FT} - w_t](1 - \chi_t) + \right. \\
&\left. [(A_t^{STW} - w_t)(1 - K_t^*) - C(K_t^*)] \chi_t - \frac{\rho_t^e}{1 - \rho_t^e} (f - \tau^f) + \frac{\kappa}{q(\theta_t)} \right\} \tag{A.17} \\
&\Leftrightarrow \\
\frac{(1 - \rho_t^e)(f - \tau^f)}{1 - \rho_t^e} &= -\left\{ [A_t^{FT} - w_t](1 - \chi_t) + \right. \\
&\left. [(A_t^{STW} - w_t)(1 - K_t^*) - C(K_t^*)] \chi_t + \frac{\kappa}{q(\theta_t)} \right\} \\
&\Leftrightarrow \\
f - \tau^f &= -\left\{ [A_t^{FT} - w_t](1 - \chi_t) + \right. \\
&\left. [(A_t^{STW} - w_t)(1 - K_t^*) - C(K_t^*)] \chi_t + \frac{\kappa}{q(\theta_t)} \right\}
\end{aligned}$$

Result of the equation (A.17) gives the value of threshold productivity. At the threshold the value of the job is equal with the separation cost. Furthermore, there are three separate cases. The first is that there is no STW, i.e.  $\chi_t = 0$ . Hence, all the separations



happen to full-time workers. The idiosyncratic productivity threshold is given by

$$\begin{aligned}
f - \tau^f &= - \left[ A_t^{FT} - w_t + \frac{\kappa}{q(\theta_t)} \right] \\
&\Rightarrow \\
f - \tau^f &= - \left[ A_t - v_t^f - w_t + \frac{\kappa}{q(\theta_t)} \right] \\
&\Leftrightarrow \\
v_t^f &= f - \tau^f + A_t - w_t + \frac{\kappa}{q(\theta_t)},
\end{aligned} \tag{A.18}$$

which corresponds the job destruction condition from the canonical search and matching model with endogenous separations.

The second case is that all workers are doing STW, i.e.  $\chi_t = 1$ . Then, the idiosyncratic productivity threshold is given by

$$\begin{aligned}
f - \tau^f &= - \left[ (A_t^{STW} - w_t)(1 - K_t^*) - C(K_t^*) + \frac{\kappa}{q(\theta_t)} \right] \\
&\Rightarrow \\
f - \tau^f &= - \left[ (A_t - v_t^f - w_t)(1 - K_t^*) - C(K_t^*) + \frac{\kappa}{q(\theta_t)} \right] \\
&\Leftrightarrow \\
v_t^f &= A_t - w_t + \frac{1}{1 - K_t^*} \left[ f - \tau^f - C(K_t^*) + \frac{\kappa}{q(\theta_t)} \right],
\end{aligned} \tag{A.19}$$

which equals the threshold equation derived in Balleer et al. (2016). The equation (A.19) also gives the separation threshold when making the additional assumption that endogenously separated workers are always those who are in STW.

The third case is that a fraction of workers are in short-time work and the rest in work full time, i.e.  $1 > \chi_t > 0$ . The idiosyncratic productivity threshold is then given by

$$\begin{aligned}
f - \tau^f &= - \left\{ [A_t - v_t^f - w_t](1 - \chi_t) + \right. \\
&\quad \left. [(A_t - v_t^f - w_t)(1 - K_t^*) - C(K_t^*)]\chi_t + \frac{\kappa}{q(\theta_t)} \right\}
\end{aligned} \tag{A.20}$$

## A.2 Social planner's problem

$$\begin{aligned}
U_t^{SP} = & \max_{v_t, n_t^{FT}, n_t^{STW}, K_t, v_t^k, v_t^f} E_t \sum_{t=0}^{\infty} \beta^t \left( n_t^{FT} A_t^{FT} + n_t^{STW} A_t^{STW} (1 - K_t^*) \right. \\
& \left. + b(1 - n_t) + bK_t n_t^{STW} \right. \\
& \left. - X(\chi_t) - C(K_t) n_t^{STW} - \frac{\rho_t^e}{1 - \rho_t^e} n_t f - \kappa v_t \right),
\end{aligned} \tag{A.21}$$

such that

$$\begin{aligned}
n_{t+1}^{FT} &= (1 - \rho^x) \left[ (1 - \rho_{t+1}^e)(1 - \chi_{t+1})(n_t^{FT} + q(\theta_t)v_t + n_t^{STW}) \right], \\
n_{t+1}^{STW} &= (1 - \rho^x) \left[ (1 - \rho_{t+1}^e)\chi_{t+1}(n_t^{STW} + q(\theta_t)v_t + n_t^{FT}) \right], \\
n_{t+1} &= n_{t+1}^{FT} + n_{t+1}^{STW} \\
X(\chi_t) &= \frac{c_x}{2} \chi_t^2 \\
C(\chi_t) &= \frac{1}{2} \chi_t^2,
\end{aligned} \tag{A.22}$$

The derivation of job creation condition requires first order condition with respect to vacancies  $v_t$  and employment  $n_t^{FT}$  and  $n_t^{STW}$ . Denoting  $\phi_t^{FT}$  and  $\phi_t^{STW}$  as Lagrange multipliers of the employment evolution constraints, the FOCs of  $v_t$ ,  $n_t^{FT}$  and  $n_t^{STW}$  are respectively

$$\begin{aligned}
-\kappa + \beta \phi_{t+1}^{FT} [(1 - \rho^x)(1 - \rho_t^e)(1 - \chi_t)(q(\theta_t) + \theta_t q'(\theta_t))] \\
+ \beta \phi_{t+1}^{STW} [(1 - \rho^x)(1 - \rho_t^e)\chi_t(q(\theta_t) + \theta_t q'(\theta_t))] = 0
\end{aligned} \tag{A.23}$$

$$\begin{aligned}
A_t^{FT} - b - \frac{\rho_t^e}{1 - \rho_t^e} f - \phi_t^{FT} \\
+ \beta \phi_{t+1}^{FT} (1 - \rho^x)(1 - \rho_t^e)(1 - \chi_t) + \beta \phi_{t+1}^{STW} (1 - \rho^x)(1 - \rho_t^e)\chi_t = 0
\end{aligned} \tag{A.24}$$

$$\begin{aligned}
A_t^{STW} (1 - K_t^*) - C(K_t^*) - b + K_t^* b - \frac{\rho_t^e}{1 - \rho_t^e} f - \phi_t^{STW} \\
+ \beta \phi_{t+1}^{FT} (1 - \rho^x)(1 - \rho_t^e)(1 - \chi_t) + \beta \phi_{t+1}^{STW} (1 - \rho^x)(1 - \rho_t^e)\chi_t = 0
\end{aligned} \tag{A.25}$$

Intermediate results:

$$\frac{\kappa}{q(\theta_t)(1 - \xi_t)} = \beta E_t \phi_{t+1}^{FT} (1 - \rho^x)(1 - \rho_{t+1}^e)(1 - \chi_{t+1}) + \beta \phi_{t+1}^{STW} (1 - \rho^x)(1 - \rho_{t+1}^e) \chi_{t+1},$$

$\Rightarrow$

$$\phi_t^{FT} = A_t^{FT} - b - \frac{\rho_t^e}{1 - \rho_t^e} f + \frac{\kappa}{q(\theta_t)(1 - \xi_t)}$$

$$\phi_t^{STW} = A_t^{STW} (1 - K_t^*) - C(K_t^*) + (-1 + K_t^*)b - \frac{\rho_t^e}{1 - \rho_t^e} f + \frac{\kappa}{q(\theta_t)(1 - \xi_t)} \quad (\text{A.26})$$

Combining the results to job creation condition yields

$$\begin{aligned} \frac{\kappa}{q(\theta_t)(1 - \xi_t)} = & \beta E_t \left[ (1 - \rho_{t+1}) \left\{ (1 - \chi_{t+1})(A_{t+1}^{FT} - b) \right. \right. \\ & \left. \left. + \chi_{t+1} [(1 - K_{t+1}^*)(A_{t+1}^{STW} - b) - C(K_{t+1}^*)] \right\} \right. \\ & \left. - (1 - \rho^x) \rho_{t+1}^e f + (1 - \rho_{t+1}) \frac{\kappa}{q(\theta_{t+1})(1 - \xi_{t+1})} \right]. \end{aligned} \quad (\text{A.27})$$

Deriving the share of STW workers, requires FOC w.r.t  $\chi_t$ .

Deriving  $\chi_t$

$$\begin{aligned} & -\phi_t^{FT} (1 - \rho^x)(1 - \rho_t^e)(n_{t-1}^{FT} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{STW}) + \\ & \phi_t^{STW} (1 - \rho^x)(1 - \rho_t^e)(n_{t-1}^{STW} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{FT}) - c_\chi \chi_t = 0 \\ & \Leftrightarrow \\ & c_\chi \chi_t = (\phi_t^{STW} - \phi_t^{FT})(1 - \rho^x)(n_{t-1}^{FT} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{STW}) \\ & \Leftrightarrow \\ & \chi_t = (\phi_t^{STW} - \phi_t^{FT}) \frac{n_t}{c_\chi} \end{aligned} \quad (\text{A.28})$$

Deriving  $\rho_t^e$

$$\begin{aligned}
& -\frac{fn_t}{(1-\rho_t^e)^2} - \phi_t^{FT}(1-\rho^x)(1-\chi_t)(n_{t-1}^{FT} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{STW}) \\
& \quad - \phi_t^{STW}(1-\rho^x)\chi_t(n_{t-1}^{STW} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{FT}) = 0 \\
& \Leftrightarrow \\
& \frac{fn_t}{(1-\rho_t^e)^2} = -\phi_t^{FT}(1-\rho^x)(1-\chi_t)(n_{t-1}^{FT} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{STW}) \\
& \quad - \phi_t^{STW}(1-\rho^x)\chi_t(n_{t-1}^{STW} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{FT}) \\
& \Leftrightarrow \\
& \frac{fn_t}{1-\rho_t^e} = -\phi_t^{FT}(1-\rho^x)(1-\rho_t^e)(1-\chi_t)(n_{t-1}^{FT} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{STW}) \\
& \quad - \phi_t^{STW}(1-\rho^x)(1-\rho_t^e)\chi_t(n_{t-1}^{STW} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{FT}) \tag{A.29} \\
& \Leftrightarrow \\
& \frac{fn_t}{1-\rho_t^e} = -[\phi_t^{FT}(1-\chi_t) + \phi_t^{STW}\chi_t](1-\rho^x)(1-\rho_t^e)(n_{t-1}^{STW} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{FT}) \\
& \Leftrightarrow \\
& \frac{fn_t}{1-\rho_t^e} = -[\phi_t^{FT}(1-\chi_t) + \phi_t^{STW}\chi_t]n_t \\
& \Leftrightarrow \\
& f = -[\phi_t^{FT}(1-\chi_t) + \phi_t^{STW}\chi_t](1-\rho_t^e) \\
& \Leftrightarrow \\
& \frac{f}{\phi_t^{FT}(1-\chi_t) + \phi_t^{STW}\chi_t} + 1 = \rho_t^e
\end{aligned}$$

### A.3 Proof of proposition 1

Now, in order to compare the differences in the two equations, I subtract the decentralized solution from the social planner solution with wage bargaining outcome included into the decentralized condition.

Next, the wage is replaced by its definition, for instance  $w_t^{FT} = \gamma A_t^{FT} + (1-\gamma)b$ . Using

the wage rule and after some reorganizing, the competitive solution becomes

$$\begin{aligned}
0 = & \beta E_t \left( (1 - \rho^x) \left[ (1 - \rho_{t+1}^e)(1 - \chi_{t+1})(A_{t+1}^{FT} - b) \right. \right. \\
& + (1 - \rho_{t+1}^e)\chi_{t+1}(A_{t+1}^{STW}(1 - K_{t+1}^*) - (1 - K_{t+1}^*)b - \frac{1}{(1 - \gamma)}C(K^*)) \\
& \left. \left. - \frac{1}{(1 - \gamma)}(1 - \rho^x)\rho_{t+1}^e(f - \tau^n) + (1 - \rho_{t+1})\frac{\kappa}{q(\theta_{t+1})(1 - \gamma)} \right) \right. \\
& \left. - \frac{\kappa}{q(\theta_t)(1 - \gamma)} \right).
\end{aligned} \tag{A.30}$$

The result of the subtraction becomes

$$\begin{aligned}
E_t \beta (1 - \rho^x)(1 - \rho_{t+1}) \frac{\kappa}{q(\theta_{t+1})} \left( \frac{1}{1 - \xi_{t+1}} - \frac{1}{1 - \gamma} \right) - \frac{\kappa}{q(\theta_t)} \left( \frac{1}{1 - \xi_t} - \frac{1}{1 - \gamma} \right) \\
- \left( f - \frac{f - \tau^f}{1 - \gamma} \right) \beta E_t [(1 - \rho^x)\rho_{t+1}^e] \\
- \left( c_k - \frac{c_k - \tau^K}{1 - \gamma} \right) \frac{1}{2} E_t \beta (K_{t+1}^*)^2 = 0
\end{aligned} \tag{A.31}$$

Where all the individual conditions of optimality are:  $\xi_t = \gamma$ ,  $\tau^f = \gamma f$  and  $\tau^K = \gamma c_k$ .

### A.3.1 Comparing STW conditions

Equality in hour reductions (the decentralized vs. the social planner solution)

$$\begin{aligned}
-\frac{(1-\gamma)(A_t^{STW} - b)}{c_k - \tau^K} &= -\frac{A_t^{STW} - b}{c_k} \\
&\Leftrightarrow \\
-\frac{(1-\gamma)}{c_k - \tau^K} &= -\frac{1}{c_k} \\
&\Leftrightarrow \\
-(1-\gamma)c_k &= -c_k + \tau^K \tag{A.32} \\
&\Leftrightarrow \\
-c_k + \gamma c_k &= -c_k + \tau^K \\
&\Leftrightarrow \\
\tau^K &= \gamma c_k.
\end{aligned}$$

The hour reduction conditions coincides with the other conditions related to the transfers from section above.

### A.3.2 No STW case

Applying the definitions of  $\lambda_t^{FT}$  and  $\lambda_t^{STW}$ , the decentralized condition (A.15) can be developed further as

$$\begin{aligned}
\chi_t &= (\lambda_t^{STW} - \lambda_t^{FT}) \frac{n_t}{c_\chi} \\
&\Leftrightarrow \tag{A.33} \\
\chi_t &= \left[ (A_t^{STW} - w_t)(1 - K_t^*) - C(K_t^*) - (A_t^{FT} - w_t) \right] \frac{n_t}{c_\chi}.
\end{aligned}$$

The profits of full-time workers, i.e.  $(A_t^{FT} - w_t)$  are always positive or zero. By assumption STW workers are generating deficits. Consequently, the value in brackets, comparing the output of STW and full-time workers is negative. In turn, the share of workers in STW,

i.e.  $\chi_t$  cannot be negative, hence the cost  $c_\chi < 0$ , or the definition of the share is assumed as

$$\chi_t = \max\left(\left(\lambda_t^{STW} - \lambda_t^{FT}\right) \frac{n_t}{c_\chi}, 0\right), \quad (\text{A.34})$$

and if  $c_\chi$  is set to positive value there is no STW.

### A.3.3 Separation and STW conditions

The STW share of workers from decentralized and social planner solutions are compared as  $\chi_t^{SP} = \chi_t^{decentralized}$  resulting in

$$\left(-\phi_t^{FT} + \lambda_t^{FT}\right) + \left(\phi_t^{STW} - \lambda_t^{STW}\right) = 0, \quad (\text{A.35})$$

where the equality holds for certainty when the shadow values from the two solutions are equal. The comparison of marginal values results in

$$\begin{aligned} -\phi_t^{FT} + \lambda_t^{FT} &= 0 \\ \Leftrightarrow \\ A_t^{FT} - b - \frac{\rho_t^e}{1 - \rho_t^e} f + \frac{\kappa}{q(\theta_t)(1 - \xi_t)} &= \\ (1 - \gamma)(A_t^{FT} - b) - \frac{\rho_t^e}{1 - \rho_t^e} (f - \tau^f) + \frac{\kappa}{q(\theta_t)}, \end{aligned} \quad (\text{A.36})$$

which results in the subset of the same optimality conditions as the comparison of job creation conditions, i.e.  $\xi_t = \gamma$ ,  $\tau^f = \gamma f$ .

The second shadow values compare as

$$\begin{aligned} -\phi_t^{STW(SP)} + \lambda_t^{STW} &= 0 \\ \Leftrightarrow \\ (1 - K_t^*)(A_t^{STW} - b) - C(K_t^*) - \frac{\rho_t^e}{1 - \rho_t^e} f + \frac{\kappa}{q(\theta_t)(1 - \xi_t)} &= \\ (1 - \gamma)(1 - K_t^*)(A_t^{STW} - b) - C(K_t^*) - \frac{\rho_t^e}{1 - \rho_t^e} (f - \tau^f) + \frac{\kappa}{q(\theta_t)}, \end{aligned} \quad (\text{A.37})$$

which holds when all the conditions from job creation comparison are met, i.e.  $\xi_t = \gamma$ ,  $\tau^f = \gamma f$ ,  $\tau^K = \gamma c_k$ .

Equations (A.36) and (A.37) show that the Lagrange multipliers from the two problems are equal, i.e.  $\phi_t^{FT} = \lambda_t^{FT}$  and  $\phi_t^{STW} = \lambda_t^{STW}$ , when the conditions of Proposition 1 hold. Furthermore, this implies that the same conditions are sufficient to yield the optimality of endogenous separation choice  $\rho_t^e$ .

## A.4 Wage bargaining

A labor union bargains wages for all workers. An outside option is a strike when no production is done. The union bargains a merit based wage rule such that the more productive workers ear higher wages. The Nash bargaining problem is

$$\arg \max_{w_t} (W_t - \tilde{W}_t)^\gamma (F_t - \tilde{F}_t)^{1-\gamma}, \quad (\text{A.38})$$

where

$$W_t = w_t + \beta E_t[(1 - \rho_t)W_{t+1} + \rho_{t+1}U_{t+1}], \quad (\text{A.39})$$

$$\tilde{W}_t = b_t + \beta E_t[(1 - \rho_t)\tilde{W}_{t+1} + \rho_{t+1}U_{t+1}], \quad (\text{A.40})$$

$$F_t = (A_t - \varepsilon_t) - w_t + \beta E_t \frac{\kappa}{q(\theta_{t+1})}, \quad (\text{A.41})$$

$$\tilde{F}_t = \beta E_t \frac{\kappa}{q(\theta_{t+1})}, \quad (\text{A.42})$$

Solution of A.38 is

$$(W_t - \tilde{W}_t) = \frac{\gamma}{1 - \gamma} (F_t - \tilde{F}_t), \quad (\text{A.43})$$

and inserting value functions results in

$$\begin{aligned} (w_t - b) &= \frac{\gamma}{1 - \gamma} (A_t - \varepsilon - w_t) \\ \Leftrightarrow w_t &= \gamma(A_t - \varepsilon) + (1 - \gamma)b. \end{aligned} \quad (\text{A.44})$$



Then aggregate wages for full time and short time workers become

$$\begin{aligned}
 w_t^{FT} &= \int_{-\infty}^{v_t^k} \gamma(A_t - \varepsilon)g(\varepsilon)d\varepsilon + (1 - \gamma)b \\
 &\Leftrightarrow w_t^{FT} = \gamma A_t^{FT} + (1 - \gamma)b,
 \end{aligned}
 \tag{A.45}$$

and similarly

$$w_t^{STW} = \gamma A_t^{STW} + (1 - \gamma)b.
 \tag{A.46}$$

## A.5 Alternative wage rule and its efficiency

The wage defined above internalizes job specific productivity by taking into account the idiosyncratic component. As a consequence, the idiosyncratic productivity does not give rise to social inefficiencies with this wage definition. This section discusses an alternative wage rule, in which the wage is not internalizing the job specific productivity.

In Balleer et al. (2016) the wage is constant for all workers. The wage is defined at the unconditional mean of the idiosyncratic productivity, and further, by assuming that this mean is zero. As a result, the wage depends on the aggregate productivity component, and the outside option for workers. This changes the value function of the firm to

$$F_t^{alt} = A_t - w_t + \beta E_t \frac{\kappa}{q(\theta_{t+1})},
 \tag{A.47}$$

and the wage becomes

$$w_t = \gamma A_t + (1 - \gamma)b.
 \tag{A.48}$$

Using this wage rule, the profits of a worker working full time  $\pi_t^{FT}$  are given by

$$\begin{aligned}
\pi_t^{FT} &= A_t^{FT} - w \\
&\Leftrightarrow \\
\pi_t &= A_t^{FT} - (\gamma A_t + (1 - \gamma)b) \\
&\Leftrightarrow \\
\pi_t^{FT} &= \int_{-\infty}^{v_t^k} (A_t - \varepsilon_t) g(\varepsilon) d\varepsilon - (\gamma A_t + (1 - \gamma)b) \\
&\Leftrightarrow \\
\pi_t^{FT} &= (1 - \gamma)(A_t + b) - \int_{-\infty}^{v_t^k} \varepsilon_t g(\varepsilon) d\varepsilon.
\end{aligned} \tag{A.49}$$

The same wage rule for the profits of a STW worker yields

$$\pi_t^{STW} = \left[ (1 - \gamma)(A_t + b) - \int_{v_t^k}^{v_t^f} \varepsilon_t g(\varepsilon) d\varepsilon \right] (1 - K_t^*). \tag{A.50}$$

In order to investigate the social efficiency of this alternative wage rule, the same notation as in equations above is applied to the social planner solution. Specifically, the social planner solution of output and home production for full-time and STW workers are respectively  $y_t^{FT}$  and  $y_t^{STW}$ , given by

$$y_t^{FT} = A_t + b - \int_{-\infty}^{v_t^k} \varepsilon_t g(\varepsilon) d\varepsilon. \tag{A.51}$$

and

$$y_t^{STW} = \left[ A_t + b - \int_{v_t^k}^{v_t^f} \varepsilon_t g(\varepsilon) d\varepsilon \right] (1 - K_t^*). \tag{A.52}$$

As is done in section A.3, I first apply the equations (A.49) and (A.50) in the decentralized job creation condition (A.14), in order to get a job creation condition with the functional

form of wage, as

$$\begin{aligned} \frac{\kappa}{q(\theta_t)(1-\gamma)} = & \beta E_t \left( (1-\rho_t) \left[ (1-\chi_{t+1})(A_{t+1}-b - \frac{1}{(1-\gamma)} \int_{-\infty}^{v_t^k} \varepsilon_{t+1} g(\varepsilon) d\varepsilon) \right. \right. \\ & \left. \left. + \chi_{t+1}(1-K_{t+1}^*)(A_{t+1}-b - \frac{1}{(1-\gamma)} \int_{-\infty}^{v_t^k} \varepsilon_{t+1} g(\varepsilon) d\varepsilon) - \frac{1}{(1-\gamma)} C(K^*) \right] \right. \\ & \left. - \frac{1}{(1-\gamma)} (1-\rho^x) \rho_{t+1}^e (f-\tau^n) + (1-\rho_{t+1}) \frac{\kappa}{q(\theta_{t+1})(1-\gamma)} \right). \end{aligned} \quad (\text{A.53})$$

Second, I compare this decentralized job creation condition with the social planner condition, as in section A.3, by subtracting the decentralized condition in (A.53) from the social planner condition (A.27). The subtraction yields

$$\begin{aligned} & \underbrace{E_t \beta (1-\rho_t) (1-\chi_{t+1}) \left( 1 - \frac{1}{1-\gamma} \right) \int_{-\infty}^{v_t^k} \varepsilon_{t+1} g(\varepsilon) d\varepsilon +}_{\text{Full time workers' productivity (+/-)}} \\ & \underbrace{E_t \beta (1-\rho_t) \chi_{t+1} \left( 1 - \frac{1}{1-\gamma} \right) \int_{v_t^k}^{v_t^f} \varepsilon_{t+1} g(\varepsilon) d\varepsilon}_{\text{STW workers' productivity (+/-)}} \\ & + \Lambda = 0, \end{aligned} \quad (\text{A.54})$$

in which  $\Lambda$  is the left hand side of the equation (A.31), i.e. it contains the inefficiencies from congestion externality, STW costs and separation cost. These two additional inefficiencies arise from the wage bargaining not internalizing the job specific productivity. When workers have bargaining power, the multiplier  $\frac{1}{1-\gamma} > 1$ , and the idiosyncratic component  $\varepsilon_{t+1}$  gives rise to externalities on job creation. These externalities can be positive or negative depending on the assumptions about the distribution of idiosyncratic productivity  $g(\varepsilon)$ .

For simplicity, let us make the following assumptions about the idiosyncratic distribution: i) the unconditional mean is zero, and ii) the distribution is symmetric with respect to the mean. In other words, the positive and negative values of  $\varepsilon_t$  have equal probability. We can consider normal distribution as an example, which is depicted in Figure 1 of the main text. As in the figure, the distribution of productivities for full-time workers

is the original distribution from which the endogenous separations and STW cut off the right tail. Hence, the mean  $\varepsilon_t$  for the full-time workers, i.e. conditional mean, is negative for all the positive shares of endogenous separations or STW. On the contrary, for the STW workers the mean is positive.<sup>9</sup> More precisely, the integral over the idiosyncratic component  $\varepsilon_t$  in equation (A.49) is negative and in equation (A.50) positive.

The inefficiency is the following. When the wage is constant to all workers according to (A.48), it is too low for those working full-time, and too high for those in STW, as compared with their productivities. Hence, there is a positive externality on job creation from the full-time workers' productivity, i.e. firms are creating too many new jobs from the social welfare perspective. On the contrary, there is a negative externality on job creation from the STW workers, i.e. the firms are creating too few new jobs. Which of these externalities is larger depends on the calibration of the model, most notable the shares of the workers in STW and full-time work.

## A.6 Households and the government

The household is stylized and makes consumption savings decisions to maximize utility given by

$$\max_{C_t, B_{t+1}} = \sum_{t=0}^{\infty} E_t \beta^t U(C_t) \quad (\text{A.55})$$

subject to a budget constraint given by

$$\begin{aligned} C_t + B_{t+1} &= n_t^{FT} w_t^{FT} + n_t^{STW} w_t^{STW} (1 - K_t^*) \\ &+ (1 - n_t) b + n_t^{STW} K_t^* b + (1 + R_t) B_t + \Pi_t - T_t, \end{aligned} \quad (\text{A.56})$$

in which  $C_t$  is consumption,  $B_t$  a one period risk-free government bond,  $T_t$  a lump-sum tax and  $\Pi_t$  profits from the firm.

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<sup>9</sup>Only in the extreme case in which the share of full-time workers is smaller than the endogenous separation rate, the mean for STW workers could become negative.

Resulting in a consumption Euler equations as

$$\frac{1}{R_{t+1}} = \beta E_t \frac{U'(C_{t+1})}{U'(C_t)}, \quad (\text{A.57})$$

where  $U'$  is a derivative w.r.t the control variable.

The government runs a balanced budget as

$$\tau^f + \tau^K = T_t + B_t - R_t B_{t-1}. \quad (\text{A.58})$$

## A.7 Numerical illustration

The following presents a numerical illustration of the decentralized and socially optimal economies. The impact of the transfers proposed above are studied separately, to demonstrate some of the difference between them. Notice that this numerical exercise is not based on an empirically relevant calibration, but artificial configuration which is chosen to present some key features of the model.

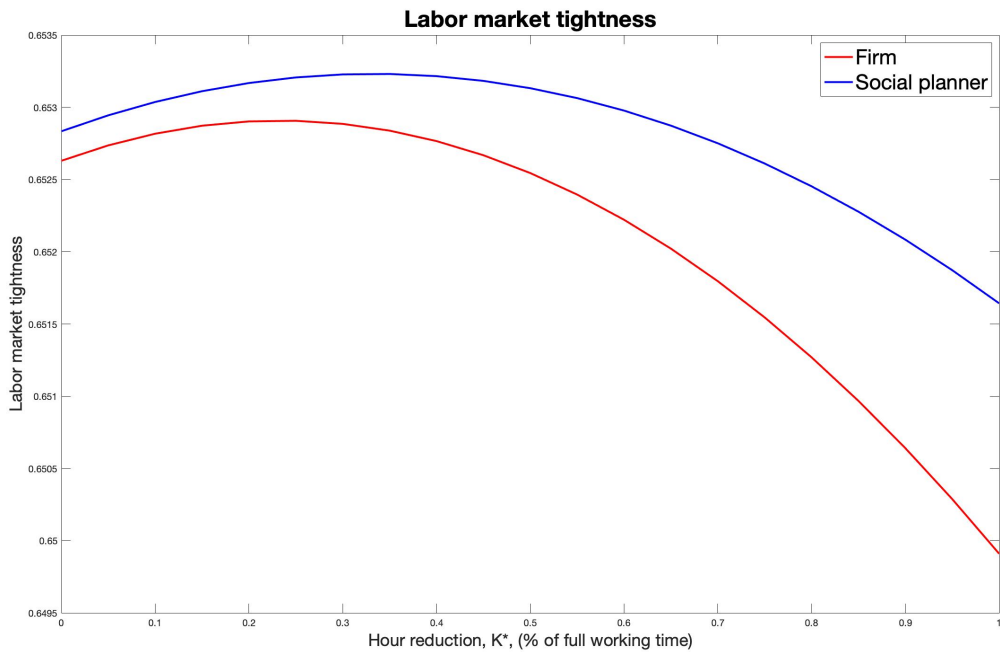


Figure 3: Labor market tightness with the different levels of hour reductions. The blue line is the social planner vacancy creation curve from equation (17) and the red line the competitive economy, i.e. equation (10). The Hosios condition is assumed to hold, but there are no transfers.

Figure 3 presents the relationship of labor market tightness and working-hour reductions in steady-state. A systematic inefficiency is shown as the curve of the competitive firm is below the social planner's curve with any level of working time. The curves are increasing at first, because the convex cost of reducing working time is small and at the same time the hour reductions are linear.

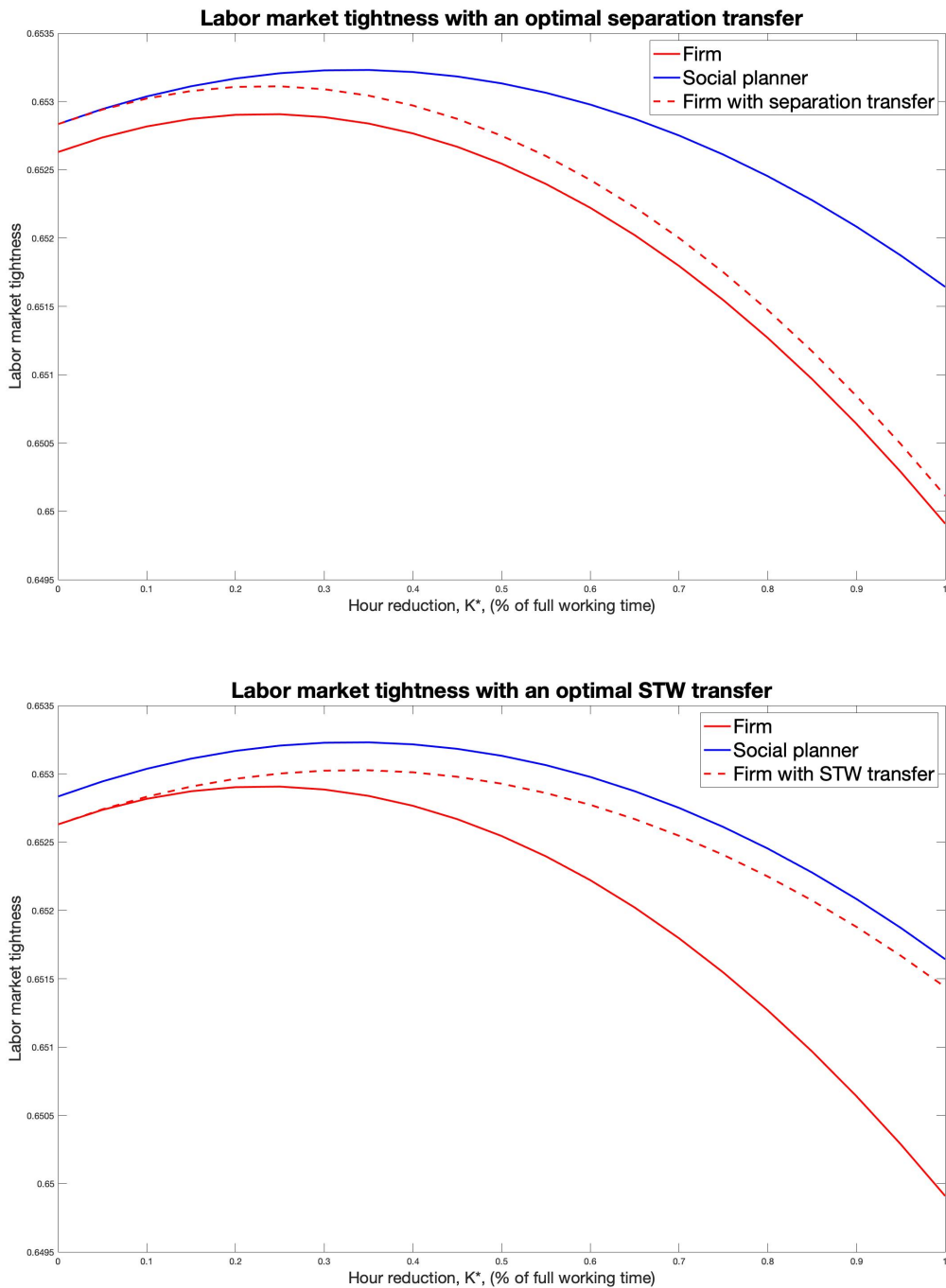


Figure 4: Labor market tightness and hour reductions with an optimal separation transfer above and an STW transfer below. The Hosios condition is assumed to hold. The blue line is the social planner vacancy creation curve from equation (17), the red line the competitive economy and the red dashed line the competitive economy with the transfer, i.e. equation (10) with  $\tau^f = \gamma f$  first and  $\tau^K = \gamma c_K$  in second.

The gains from adjusting the working time are larger than the costs from it. When the hour reduction increases, the cost increases according to a quadratic function and becomes larger than the benefit from hour reductions. This turns the curves downward sloping and in total results in a convex shape.

Figure 4 presents the comparison of optimal transfer derived in previous section. The first graph shows the impact of separation transfer. This transfer moves the competitive curve directly upward, making the decentralized economy closer to the socially optimal with any level of working time in STW. In addition, if there is no STW, i.e. the hour reduction  $K^* = 0$ , the decentralized economy is optimal. When the hour reductions are increasing, the inefficiency of the competitive economy, i.e. the distance of the curves, increases as well.

The lower part of Figure 4 shows the impact of STW transfer. The optimal STW transfer incentivizes the firm to reduce more working hours. Furthermore, there are more resources available for creating vacancies. As a result, the curve of the firm with STW transfer moves up and right, where more vacancies are created and the level of STW is increased. However, when the hour reduction  $K^*$  is small, the impact of the transfer on job creation is negligible since the cost is also small.

The cases in Figure 4 illustrate the difference in channels through which the transfers affect labor markets. The separation transfer reduces the cost of separations, which increases the total profits of the firm. A fraction of these profits is used to increase vacancy postings. The cost of hour reduction by the firm does not depend on the separation cost, hence the optimal choice of the working time in STW is unaffected. The STW transfer subsidizes the hour reductions and incentivizes firms to increase the level of STW. There are two consequences. The first is analogical with the separation transfer, in which the cost reduction increases profits. The second is the decreased working time of low productive workers, which decreases losses generated by these workers, allowing larger surplus to be used in vacancy creation.



## B Appendix, Calibration

### B.1 Data

The observables used in calibration are retrieved from the Bundesbank website (<https://www.bundesbank.de/en/statistics>). The following lists the data series. The period considered is 2000-2021, and the steady-state targets in the calibration exercise are calculated as simple averages over the sample period.

**Unemployment rate.** Unemployment registered pursuant to section 16 Social Security Code III / Germany / Social Security Code III and Social Security Code II / Rate / Calendar and seasonally adjusted

Series: BBDL1.M.DE.Y.UNE.UBA000.A0000.A01.D00.0.R00.A

Source: Seasonal adjustment based on data provided by the Federal Employment Agency.

**Vacancies.** Reported vacancies, total / Germany / Total / Absolute value / Calendar and seasonally adjusted

Series: BBDL1.M.DE.Y.VAC.VBA000.A0000.A00.D00.0.ABA.A

Source: Seasonal adjustment based on data provided by the Federal Employment Agency.,

**Employed workers.** Employed persons according to ESA 2010 / Germany / Domestic concept / Absolute value / Calendar and seasonally adjusted

Series: BBDL1.M.DE.Y.EMP.EAA000.A0000.A00.D10.0.ABA.A

Source: Seasonal adjustment based on data provided by the Federal Statistical Office.

**Short-time workers.** Short-time workers, basis for entitlement according to section 96 only / Germany / Social Security Code III / Absolute value / Calendar and seasonally adjusted

Series: BBDL1.M.DE.Y.LMP.LKA100.A0000.A02.D00.0.ABA.A

Source: Seasonal adjustment based on data provided by the Federal Employment Agency.,